ıges) **Reg. No. :** ....

›de No. : 30566 E **Sub. Code : AMMA 21 / CMMA 21**

c. (CBCS) DEGREE EXAMINATION, APRIL 2022.

Second Semester

Mathematics — Core

:FFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2020 onwards)

::e : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

The complete solution of $y = px + p^2$ where $\left(p = \frac{dy}{dx}\right)$ is ————————.

(a) $y = x^2 + c$ (b) $y = cx^2 - c$

(c) $y = cx + c^2$ (d) $cx - c$

2. The general solution of $(D^2 - 4)y = 0$ is $y =$ ————.

(a) $Ae^{2x} + Be^{-2x}$ (b) $Ae^{4x} + Be^{-4x}$

(c) $Ae^{3x} + Be^{x}$ (d) $Ae^{4x} + B$

3. The particular integral of $(D^2 - 9)y = \cos 3x$ is ————————

(a) $\frac{\cos 3x}{18}$ (b) $\frac{\cos 3x}{9}$

(c) $\frac{\cos 3x}{-18}$ (d) 0

4. The solution of the differential equation $p^2 - 9p + 18 = 0$ where $p = \frac{dy}{dx}$ is ————

(a) $(y - 3x - c)(y - 6x - c) = 0$

(b) $(y - 6y - c)(y - 3x - c) = 0$

(c) $(x - 6y - c)(3x - y - c) = 0$

(d) $x^2 - 9x + 18 = 0$

5. The direction ratios of the line joining (1,2,–1) and (2, –1, 1) are ————————.

(a) 2, 6, 4 (b) 1, –3, 2

(c) –2, –6, –4 (d) –1, 3, –2

The equation of the plane through (1,0,2) and parallel to the plane $2x + 3y - 4z = 0$ is ————

(a) $3x + 2y - 3z + 6 = 0$ (b) $3x + 2y - 3z - 6 = 0$

(c) $2x + 3y - 4z + 6 = 0$ (d) $2x + 3y - 4z - 6 = 0$

The line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is parallel to the plane ————

(a) $x - 2y - 4z + 7 = 0$ (b) $2x - 2y - 4z + 7 = 0$

(c) $x - 7y - 4z + 7 = 0$ (d) $7x - 7y - 4z + 7 = 0$

If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ ———— the plane $ax + by + cz + d = 0$ then $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d = 0$.

(a) lies in (b) is parallel to

(c) bisects (d) is proportional to

Centre and radius of the sphere $x^2 + y^2 + z^2 - 6x - 2y - 4z - 11 = 0$ is

(a) (0,2,4) and 16 (b) (0,–1,2) and –4

(c) (3,1,2) and 5 (d) (1,–1,2) and –6

The equation of the tangent plane at the origin to the sphere $x^2+y^2+z^2-8x-6x+4z=0$ is ————

(a) $4x + 3y - 2z = 0$ (b) $4x - 3y - 2z = 0$

(c) $4x - 3y + 2z = 0$ (d) $-4x + 3y + 2z = 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve: $x^2(y - px) = yp^2$.

Or

(b) Solve: $p^2 + 2py\cot x - y^2 = 0$.

12. (a) Solve: $(D^3 - 3D^2 + 3D - 1)y = x^2e^x$.

Or

(b) Solve: $x^2y'' + 3xy' + y = \frac{1}{(1-x)^2}$.

13. (a) The line joining A(4,3,2) and B(1,2,–3) meets the planes YOZ, XOY in C,D respectively. Find the coordinates of C and and D and the ratios in which they divide AB.

Or

(b) Find the equation of the plane through the line of intersection of the plane $2x + y + 3z - 4 = 0$ and $4x - y + 5z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z + 6 = 0$.

14. (a) Find the perpendicular distance form (3,9,–1) to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$

Or

(b) Find the equations of the plane passing through the line $5x-2y+7=0=x-3y+z-4$ and parallel to the line $\frac{x}{2}=\frac{y}{1}=\frac{z-1}{-2}$.

15. (a) Find the equation to the sphere through the four points (0,1,3), (1,2,4), (2,3,1) and (3,0,2)

Or

(b) Find the equation of the tangent line to the circle $x^2+y^2+z^2-x+4z=0$; $3x-2y+4z+1=0$ at the point (1,–2,–2).

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve: $\frac{dx}{dt}-\frac{dy}{dt}+x-y=1$

$2\frac{dx}{dt}+\frac{dy}{dt}=t$

Or

(b) Solve: $(px-y)(x+yp)=a^2p\ (x^2=x,\ y^2=y)$.

17. (a) Solve: $(D^2+1)y=x^2e^2x+x\cos x$.

Or

(b) Apply the method of variation of parameters to solve $y''=3y'=2y=x^2$

18. (a) A line makes an angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$.

Or

(b) Show that the origin lies in the acute angle between the planes $x+2y+2z=9,\ 4x-3y+12z+13=0$. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

19. (a) Find the equations of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane $2x-3y+2z+3=0$.

Or

(b) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their common point and find the equation of the plane which they lie.

20. (a) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0\ x+2y+3z=8$ and touches the plane $4x+3y=25$

Or

(b) Show that the conditions for the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ to cut the sphere $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ in a great circle is $2uu_1+2vv_1+2ww_1-(d+d_1)=2r_1^2$ where r_1 is the radius of the latter sphere.

———————

(6 pages)

Reg. No. : ....

Code No. : 20291 E **Sub. Code : AMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fourth Semester

Mathematics — Core

ABSTRACT ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is not a symmetric relation on $S = \{a,b,c,d\}$?

   (a) $\{(a,b),(b,a)\}$

   (b) $\{(a,b),(b,c),(a,c)\}$

   (c) $\{(a,a),(b,b)\}$

   (d) $\{(a,b),(b,c),(b,a),(c,b)\}$

2. If the order of an element α is a group G is x then the order of the element α^{-1} is

   (a) -1 (b) $-x$

   (c) x (d) x^{-1}

3. In the group $G = \{1,-1,i,-i\}$ with usual multiplication, the inverse of i is ————

   (a) 1 (b) i

   (c) $-i$ (d) -1

4. Let G be a finite group and H be a subgroup of G. If $[G:H] = |G|$ then H is ————

   (a) $\{e\}$ (b) G

   (c) H (d) e

5. If $f : G \to G'$ is 1-1, then $O(\ker f) =$ ————

   (a) -1 (b) 0

   (c) 1 (d) 2

6. $R^+/\{1,-1\} \cong$ ————

   (a) R^+ (b) R^-

   (c) R (d) $\{1,-1\}$

7. In the ring $(R,+,\cdot)$ the set of units is ————

   (a) Z (b) $\{1,-1\}$

   (c) $R-\{0\}$ (d) R

8. In the ring $M_2(R)$, the unit element is

   (a) $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ (b) $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

   (c) $\begin{pmatrix}2 & 4\\ 2 & 4\end{pmatrix}$ (d) $\begin{pmatrix}4 & 2\\ 2 & 4\end{pmatrix}$

9. Which one is a prime ideal in R?

   (a) (-1) (b) (0)

   (c) (1) (d) (2)

10. If $f(x), g(x) \in Z_4[x]$ be defined as $f(x) = x^2 + 3x + 1$ and $g(x) = 2x^2 + x$ then degree of $f(x).g(x)$ is ————

    (a) 3 (b) 4

    (c) 2 (d) 1

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Prove that the set of all equivalence classes determined by an equivalence relation defined on a set S forms a partition on the set S.

    Or

    (b) If $f : A \to B$, $g : B \to C$ are bijections, prove that $gof : A \to C$ is also a bijection.

12. (a) If G is a finite group with even number of elements then prove that G contains at least one element of order 2.

    Or

    (b) Let A and B be subgroups of a finite group G such that A is a subgroup of B. Show that $[G:A] = [G:B][B:A]$.

13. (a) Prove that every subgroup of a cyclic group is cyclic.

    Or

    (b) If $f : G \to G'$ is a group homomorphism prove that f is $1-1 \Leftrightarrow \ker f = \{e\}$.

14. (a) Prove that a finite commutative ring R without zero - divisors is a field.

Or

(b) Show that the only ideals of a field F are F and $\{0\}$.

15. (a) Show that z_n is an integral domain if and only if n is prime.

Or

(b) Prove that every finite integral domain is a field.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Let A and B be two subgroups of a group G. Prove that AB is a subgroup of G if and only if $AB = BA$.

Or

(b) Prove that the union of two subgroups of a group G is a subgroup if and only if one is contained in the other.

17. (a) Let H and K be two finite subgroups of a group G. Prove that $|HK| = \frac{|H||K|}{|H \cap K|}$.

Or

(b) State and prove Lagrange's theorem.

18. (a) If $f : G \to G'$ is a homomorphism with Kernel K, prove that $\frac{G}{K} \cong f(G)$.

Or

(b) State and prove Cayley's theorem.

19. (a) Let R be a commutative ring with identity prove that an ideal M of R is a maximal ideal $\Leftrightarrow R/M$ is a field.

Or

(b) Prove the following

(i) z_n is an integral domain $\Leftrightarrow n$ is a prime number.

(ii) the characteristics of an integral domain is either 0 or a prime number.

20. (a) State and prove division algorithm.

Or

(b) Prove that every integral domain can be embedded in a field.

(8 pages) **Reg. No. :** ....

Code No. : 20292 E **Sub. Code : AMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics – Core

LINEAR ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Which of the following is a subspace of a vector space R^3?
   (a) $W = \{(a, 0, 0)/a \in R\}$
   (b) $W = \{Ka, Kb, Kc/K \in R\}$
   (c) $W = \{(a, a+1, 0)/a \in R\}$
   (d) $W = \{(a, 0, b)/a, b \in R\}$

2. Let V be a vector space over a field F and W, a subspace of V. If $T: V \to \frac{V}{W}$ defined by $T(V) = W + V$ is a linear transformation, $\ker T =$ ————.
   (a) $\{0\}$ (b) V
   (c) $\{1\}$ (d) W

3. If $S = \{(2, 0)\}$ in $V_2(R)$ then $L(S) =$ ————.
   (a) $\{(x, 0)/x \in R\}$ (b) $\{(0, x)/x \in R\}$
   (c) $\{(0, 0)\}$ (d) $\{(0, 2)\}$

4. The vectors (a, b) and (c, d) are linearly dependent iff ————.
   (a) $ab - cd = 0$ (b) $ac - db = 0$
   (c) $ab - bc = 0$ (d) $ad - bc = 0$

5. $T: V_2(R) \to V_2(R)$ given by $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ with respect to the standard basis then the linear transformation is ————.
   (a) $T(a, b) = (a\sin\theta + b\cos\theta, -a\cos\theta + b\sin\theta)$
   (b) $T(a, b) = (a\cos\theta + b\sin\theta, -a\sin\theta + b\cos\theta)$
   (c) $T(a, b) = (-a\sin\theta + b\cos\theta, a\cos\theta + b\sin\theta)$
   (d) $T(a, b) = (-a\cos\theta + b\sin\theta, a\sin\theta + b\cos\theta)$

6. The norm of the vectors in $V_3(R)$ with standard inner product (3, –4, 0) is ————.
   (a) 3 (b) 0
   (c) 5 (d) –4

7. The rank of the matrix is $\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -3 \\ 6 & -1 & 1 \end{bmatrix}$ is ————.
   (a) 2 (b) 6
   (c) –2 (d) –3

8. If $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ then $|A| =$ ————.
   (a) 0 (b) 2
   (c) 4 (d) 1

9. For what value of k is 3 a characteristic root of $\begin{pmatrix} 3 & 1 & -1 \\ 3 & 5 & -k \\ 3 & k & -1 \end{pmatrix}$.
   (a) 5 (b) 2
   (c) –1 (d) 3

10. The eigen values of $\begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$ are
    (a) 3, 4, 1 (b) 3, 5, 3
    (c) 3, 0, 0 (d) 1, 1, 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) (i) Prove that the intersection of two subspaces of a vector space is a subspace.

    (ii) Prove that the union of two subspaces of a vector space need not be a subspace.

    Or

    (b) Let V be a vector space over a field F. A non-empty subset W of V is a subspace of V iff $u, v \in W$ and $\alpha, \beta \in F \Rightarrow \alpha u + \beta v \in W$.

12. (a) Prove that any subspace of a linearly independent set is linearly independent.

Or

(b) Prove that $S = \{(1, 0, 0), (0, 1, 0), (1, 1, 1)\}$ is a basis for $V_3(R)$.

13. (a) Let V be the set of all continuous real valued functions defined on the closed interval $[0, 1]$. Prove that V is a real inner product space with inner product defined by

$$\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$$

Or

(b) Let V be a finite dimensional inner product space. Let W be a subspace of V. Prove that $(W^{\perp})^{\perp} = W$.

14. (a) Show that the non-singular matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$. Hence evaluate A^{-1}.

Or

(b) State and prove Cayley-Hamilton theorem.

15. (a) Let f be the bilinear form defined by $V_2(R)$ by $f(x, y) = x_1 y_1 + x_2 y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f w.r.t. the basis $\{(1, 1), (1, 2)\}$.

Or

(b) Find the characteristic roots of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let V and W be vector spaces over a field F. Let $L(V, W)$ represent the set of all linear transformations from V to W. Then $L(V, W)$ itself is a vector space over F under addition and scalar multiplication defined by $(f + g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.

Or

(b) State and prove Fundamental theorem of Homomorphism.

17. (a) Let V be a finite dimensional vector space over a field F. Let W be a subspace of V. Prove that

(i) $\dim W \leq \dim V$

(ii) $\dim \dfrac{V}{W} = \dim V - \dim W$.

Or

(b) Let V be a vector space over a field F. Let $S, T \subseteq V$, then prove that

(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$

(iii) $L(S) = S \Leftrightarrow S$ is a subspace of V.

18. (a) Let V be the vector space of polynomials with inner product given by $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. Let $f(t) = t + 2$ and $g(t) = t^2 - 2t - 3$. Find

(i) $\langle f, g \rangle$

(ii) $\|f\|$.

Or

(b) Show that every finite dimensional inner product space has an orthonormal basis.

19. (a) Verify whether the following system of equations is consistent. If it is consistent find the solution

$$\begin{aligned} x - 4y - 3z &= -16 \\ 4x - y + 6z &= 16 \\ 2x + 7y + 12z &= 48 \\ 5x - 5y + 3z &= 0. \end{aligned}$$

Or

(b) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ using Cayley-Hamilton theorem.

20. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

Or

(b) Reduce the quadratic form $2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$ to the diagonal form using Lagrange's method.

———————

(6 pages) **Reg. No. :** ....

Code No. : 20293 E **Sub. Code : AMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In [0, 1] with usual metric, $B\left(0, \frac{1}{4}\right)$ is ————.

   (a) $\left(-\frac{1}{4}, \frac{1}{4}\right)$ (b) $\left[0, \frac{1}{4}\right]$

   (c) $\left[0, \frac{1}{4}\right)$ (d) $\left(0, \frac{1}{4}\right)$

2. Which of the following subsets of R is not open?

   (a) (0, 1) (b) ϕ

   (c) $(1, 2)\cup(3, 4)$ (d) Q

3. $f : M_1 \to M_2$ is continuous if and only if

   (a) $x_n - x = 0 \Rightarrow f(x_n) - f(x) = 0$

   (b) $x_n \to x \Rightarrow f(x_n) = f(x)$

   (c) $(x_n) \to x \Rightarrow (f(x_n)) \to f(x)$

   (d) $x_n - x \to 0 \Rightarrow f(x_n - x) \to 0$

4. The function $f : (0, 1) \to R$ defined by $f(x) = \frac{1}{x}$ is

   (a) not continuous

   (b) uniformly continuous

   (c) not uniformly continuous

   (d) neither continuous nor uniformly continuous

5. If $A = (0, 1] \subseteq R$, then $\overline{A}$ is ————.

   (a) (0, 1) (b) [0, 1]

   (c) (0, 1] (d) [0,1)

6. A connected subset of R is

   (a) $[4, 7]\cup[8, 10]$ (b) $[4, 6]\cup[5,7]$

   (c) $[4, 7)\cup(7, 8)$ (d) Q

7. $\bigcup_{n=1}^{\infty}[0, n) = ?$

   (a) $[0,\infty]$ (b) $(0,\infty)$

   (c) $[0, \infty)$ (d) $(0,\infty]$

8. A compact subset of R is ————.

   (a) $[0, \infty)$ (b) (3, 4)

   (c) Q (d) [1, 2.8]

9. $\bigcup_{n=1}^{\infty}\left(0, \frac{1}{n}\right) = ?$

   (a) (0,1) (b) ϕ

   (c) {0} (d) (0, 1]

10. In $R\times R$, $\overline{Q\times Q}$ is ————.

    (a) ϕ (b) Q^2

    (c) $R\times R$ (d) $Z\times Z$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In any metric space prove that each open ball is an open set.

    Or

    (b) Prove that $\overline{A\cup B} = \overline{A}\cup\overline{B}$.

12. (a) Show that the function $f : R \to R$ defined by

    $$f(x) = \begin{cases} 0, \text{ if } x \text{ is irrational} \\ 1, \text{ if } x \text{ is rational} \end{cases}$$

    is not continuous.

    Or

    (b) Prove that $f : M_1 \to M_2$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq M_1$.

13. (a) If A is a connected subset of the metric space M. Prove that $\overline{A}$ is connected.

    Or

    (b) Show that the continuous image of a connected metric space is connected.

14. (a) Prove that continuous image of a compact metric space is compact.

    Or

    (b) If A is a compact subset of a metric space (M,d), prove that A is closed.

15. (a) Let A be a subset of a metric space M. If A is totally bounded, show that A is bounded.

Or

(b) Show that a metric space is compact if and only if any family of closed sets with finite intersection property has non empty intersection.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Cantor's interaction theorem.

Or

(b) State and prove Baire's category theorem.

17. (a) (i) Let (M, d) be a metric space. Let $a \in M$, show that the function $f : M \to R$ defined by $f(x) = d(x, a)$ is continuous.

(ii) Let (M, d) be any metric space. Let $f : M \to R$, $g : M \to R$ be two continuous functions. Prove that $f + g$ is continuous.

Or

(b) Prove that $f : R \to R$ is continuous at $a \in R$ f and only if $w(f, a) = 0$.

18. (a) Prove that R is a connected metric space.

Or

(b) (i) If A and B are connected subsets of a metric space M and $A \cap B = \phi$. Prove that $A \cup B$ is a connected set.

(ii) State and prove the Intermediate value theorem.

19. (a) State and prove Heine Borel Theorem.

Or

(b) Let (M_1, d_1) be a compact metric space and (M_2, d_2), be any metric space. If $f : M_1 \to M_2$ is continuous, prove that f is uniformly continuous on M.

20. (a) If A is a totally bounded set. Prove that $\overline{A}$ is also totally bounded.

Or

(b) Prove that the metric space M is compact iff any family $\{A_\alpha\}$ of closed sets with finite intersection property has non empty intersection.

———————

(8 pages)

Reg. No. : ....

Code No. : 20381 E **Sub. Code : CAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The n^{th} degree equation $f(x)=0$ cannot have more than ———— roots

   (a) 4 (b) 6

   (c) 7 (d) n

2. If α, β, γ are the roots of the equation $x^4+px^3+qx^2+rx+5=0$ then $\sum\alpha\beta\gamma =$ ————

   (a) $-p$ (b) q

   (c) $-r$ (d) s

3. After removing the fractional coefficients from the equation $x^3-\frac{1}{4}x^2+\frac{1}{3}x-1=0$ we get ————

   (a) $x^3-1=0$

   (b) $12x^3-3x^2+4x-12=0$

   (c) $x^3-3x^2+48x-1728=0$

   (d) $x^3-3x^2+48x-1=0$

4. How many imaginary roots will occur for the equation $x^7-3x^4+2x^3-1=0$?

   (a) atmost four

   (b) exactly four

   (c) atleast four

   (d) none of these

5. The characteristic equation of $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$ is ______

(a) $\lambda^2 - 2\lambda - 1 = 0$

(b) $\lambda^2 + 2\lambda - 1 = 0$

(c) $\lambda^2 - 2\lambda + 1 = 0$

(d) $\lambda^2 + 2\lambda + 1 = 0$

6. Two eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are –2 and 3, the third eigen value is ______.

(a) 4 (b) 5

(c) 6 (d) –1

7. The Clairauts equation is ______

(a) $y = cx + f(c)$

(b) $y = px + f(p)$

(c) $\frac{dy}{dx} = \left\{p + x\frac{dp}{dx}\right\} + f'(p)\frac{dp}{dx}$

(d) none of these

8. The partial differential equation obtained from $Z = ax + by + a^2$ by eliminating the arbitrary constants 'a' and 'b' is ______

(a) $Z = px + py + a^2$ (b) $Z = qx + py + a^2$

(c) $Z = px + qy + a^2$ (d) none of these

9. $L(x) =$ ______

(a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$

(c) $-\frac{1}{s^2}$ (d) none of these

10. $L^{-1}\left[\frac{1}{s-a}\right] =$ ______

(a) 1 (b) x

(c) e^{ax} (d) e^{-ax}

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve $x^4 + 2x^2 - 16x + 77 = 0$ given that one of its root is $-2 = i\sqrt{7}$.

Or

(b) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in Harmonic progression.

12. (a) Diminish the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and hence solve the original equation.

Or

(b) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

13. (a) Find the eigen value and eigen vectors of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

Or

(b) Find the inverse of matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$.

14. (a) Form the partial differential equation by eliminate arbitrary constants 'a' and 'b' from $\log(az - 1) = x + ay + b$.

Or

(b) Form a partial differential equation by eliminating arbitrary functions 'ϕ' from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$.

15. (a) Find $L(\sin 2t \sin 3t)$.

Or

(b) (i) Prové that $L\left[e^{-ax}\right] = \frac{1}{s + a}$

(ii) If $L[f(x)] = F(s)$ then prove that $L[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in arithmetic progression if $2q^3 + 27p^2s = 9pqr$.

Or

(b) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

17. (a) Find by Horner's method, the positive root of $x^3 - 3x + 1 - 0$ lies between 1 and 2, Calculate it to three place of decimals.

Or

(b) Obtain by Newtons method, the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

18. (a) Find the eigen value and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

Or

(b) Verify Cayley–Hamilton theorem for $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

19. (a) Solve $x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y)$.

Or

(b) Solve $xp^2 - 2py + x = 0$.

20. (a) Find $L^{-1}\left[\dfrac{s^2 - s + 2}{s(s-3)(s+2)}\right]$.

Or

(b) Find $L^{-1}\left[\dfrac{cs + d}{(s+a)^2 + b^2}\right]$.

(7 pages) **Reg. No. :** ....

Code No. : 20383 E **Sub. Code : CAMA 21**

B.Sc.(CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second/Fourth Semester

Mathematics — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The unit normal vector to the surface $x^3 - xyz^3 + z^3 = 1$ at $(1, 1, 1)$ is ————

(a) $\frac{2\vec{i} - \vec{j} + 2\vec{k}}{3}$ (b) $2\vec{i} - \vec{j} + 2\vec{k}$

(c) $\frac{\vec{i} - 2\vec{j} + 2\vec{k}}{3}$ (d) $\vec{i} + 2\vec{j} + 3\vec{k}$

2. If $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ solenoidal then the value of 'a' is ————

(a) 2 (b) −2

(c) 1 (d) 0

3. The value of $\int_0^1\int_0^1 (x^2+y^2)dxdy$ is ————

(a) $\frac{2}{3}$ (b) 2

(c) $\frac{1}{2}$ (d) 2

4. The value of $\int_0^a\int_0^a\int_0^a dzdydx$ is

(a) a^3 (b) a^2

(c) a (d) 1

5. $\int_0^{\pi/2} \left(3\sin x\vec{i} + 2\cos x\vec{j}\right) dx =$ ————

(a) $3\vec{i} + 2\vec{j}$ (b) $3\vec{i} - 2\vec{j}$

(c) $-3\vec{i} + 2\vec{j}$ (d) $-3\vec{i} - 2\vec{j}$

6. If S is the sphere $x^2+y^2+z^2=1$, the value of $\iint_S \vec{r}.\hat{n}ds$ is

(a) $\frac{4\pi}{3}$ (b) 3π

(c) 4π (d) 2π

7. If C is the circle $x=\cos\theta$, $y=\sin\theta$ then $\int_C (xdy-ydx)=$ ————————

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) $\frac{\pi}{4}$

8. $\iint_C \vec{F}.d\vec{r}=\iint_S (\nabla\times\vec{F})\hat{n}\,ds$ is

(a) Fundamental theorem

(b) Gauss-divergence theorem

(c) Green's theorem

(d) Stoke's theorem

9. If $f(x)=\begin{cases}1, & 0<x<\pi \\ 0, & \pi<x<2\pi\end{cases}$ then the value of the Fourier coefficinet a_n is

(a) 0 (b) –2

(c) –3 (d) –4

10. If $f(x)=|x|$ in $(-\pi,\pi)$ then the fourier coefficient a_0 is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) $\frac{\pi^2}{2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Obtain the directional derivative of $\phi=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$

Or

(b) Prove that curl grad $\phi=\nabla\times\nabla\phi=0$.

[P.T.O.]

12. (a) Evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ using change of order of integration.

Or

(b) Evaluate $\int_0^{\log a} \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$

13. (a) Evaluate $\int_C \vec{f}.d\vec{r}$ where $\vec{f} = (x^2+y^2)\vec{i} + (x^2-y^2)\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1,1)

Or

(b) Evaluate $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ for a vector S $x^2+y^2+z^2 = a^2$ in the upper hemisphere and $z \geq 0$, $\iint_S (\nabla \times \vec{F})\hat{n}\,ds$.

14. (a) Use Green's theorem to evaluate $\int_C (x^2 y\,dx + y^3\,dy)$. where C is the closed path formed by $y = x$ and $y = x^3$ from (0,0) to (1,1).

Or

(b) Evaluate $\int_C (e^x dx + 2y\,dy - dz)$, by using stoke's theorem where C is the curve $x^2+y^2=4$, $z=2$.

15. (a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$

Or

(b) Find a sine series for $f(x) = c$ in the range 0 to π

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that $\nabla \times (\nabla \times \vec{f}) = \nabla(\nabla.\vec{f}) - \nabla^2 f$

Or

(b) If $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. If $\vec{F} = \nabla\phi$ then find the value of ϕ.

17. (a) Find the area of the region D bounded by the parabolas $y = x^2$ and $x = y^2$.

Or

(b) Evaluate $\iint_D x^2 y^2\,dx\,dy$, where D is the circular disc $x^2+y^2 \leq 1$.

18. (a) Find the work done by the force $\vec{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ along the curve $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.

Or

(b) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$

19. (a) Verify Gauss theorem for $\vec{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken over the cuboid $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$

Or

(b) Verify stoke's theorem for $\vec{F} = (x^2 - y^2)\bar{i} + 2xy\bar{j}$ int he rectangular region $x = 0$, $x = a$, $y = 0$, $y = b$.

20. (a) Express $f(x) = \frac{1}{2}(\pi - x)$ as a fourier series with period 2π, to be valid in the interval $(0, 2\pi)$

Or

(b) Find a cosine series in the range 0 to π for

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$$

(8 pages) **Reg. No. :** ....

Code No. : 20382 E **Sub. Code : CAST 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First/Third Semester

Mathematics – Allied

STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The value of μ_3' is ————

(a) $\mu_3 + 3\mu_2\mu_1' + \mu_1'^3$

(b) $\mu_3 - 3\mu_2\mu_1' + \mu_1'^3$

(c) $\mu_3 + 2\mu_2\mu_1' + \mu_1'^3$

(d) $\mu_3 - 2\mu_2\mu_1' + \mu_1'^3$

2. ———— is not affected by change of origin but affected by change of scale.

(a) arithmetic mean

(b) median

(c) moment

(d) mode

3. If $\sum(x-\bar{x})^2 = 60$, $\sum(y-\bar{y})^2 = 90$, $\sum(x-\bar{x})(y-\bar{y}) = 45$, then the correlation coefficient between the variables x and y is ————

(a) 0.6125 (b) 0.1265

(c) 0.5623 (d) 0.2516

4. If X and Y are uncorrelated, $\text{cov}(X,Y) =$ ————

(a) 0 (b) 1

(c) –1 (d) ∞

5. If $Q = 0$, then $Y =$ ————

(a) 1 (b) 0

(c) –1 (d) $-\infty$

6. For any given three attributes, the total number of positive class frequencies is ———

(a) n^2 (b) n^3

(c) 3^n (d) 2^n

7. The value for c for the probability density function $f(x) = \frac{x}{c}$, $x = 1, 2, 3, 4, 5$ is ———

(a) $\frac{1}{15}$ (b) 15

(c) $\frac{15}{2}$ (d) 1

8. If X is the number on a die when it is thrown, $E(X) =$ ———

(a) $\frac{1}{6}$ (b) 1

(c) 7 (d) $\frac{7}{2}$

9. If the mean of a Poisson distribution is λ, standard deviation = ———

(a) λ (b) λ^2

(c) $\sqrt{\lambda}$ (d) $\sqrt{\lambda^2 + \lambda}$

10. In a normal distribution, Q.D = ——— S.D.

(a) $\frac{4}{5}$ (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{5}{4}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In a frequency distribution, Bowley's coefficient of skewness is 0.6, sum of the upper and lower quartiles is 100, median is 38. Find the value of the upper quartile.

Or

(b) Fit a straight line to the following data :

X	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

12. (a) Prove that $-1 \leq \gamma \leq 1$.

Or

(b) From the following table, find the rank correlation coefficient between the height and weight.

Height (in cm)	165	167	166	170	169	172
Weight (in kg)	61	60	63.5	63	61.5	64

13. (a) Is there any inconsistency in the data given below $N = 600$; $(A) = 300$; $(B) = 400$; $(AB) = 50$.

Or

(b) Show that the relation between Yule's coefficient Q and the coefficient of colligation Y is $Q = \frac{2Y}{1+Y^2}$.

14. (a) If $f(x) = \begin{cases} Ax & for\ 0 < x < 5 \\ A(10-x) & for\ 5 \le x < 10 \\ 0 & otherwise \end{cases}$, is the probability density function of a random variable X, find the value of A.

Or

(b) If the random variable X has the following probability law $P(X = x) = q^{x-1} \cdot p$, $x = 1, 2, 3, \ldots$, find the moment generating function of X.

15. (a) If the mean of a normal distribution is 4 and the variance is 3, find its mode.

Or

(b) A book of 500 pages contains 500 mistakes. Find the probability that there are atleast four mistakes in a randomly selected page.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Calculate the first three central moments for the following data :

x	3	6	10	15	20	23	24
f	2	5	15	21	16	13	4

Or

(b) Fit a curve $y = ae^{bx}$ for the following data :

x	0	1	2	3
y	3	8	25	74

17. (a) Find the correlation coefficient from the following data :

x	65	66	67	67	68	69	70	71
y	67	68	65	68	72	72	69	71

Or

(b) Let x, y be two variables with standard deviations σ_x and σ_y respectively. If $u = x + ky$, $v = x + \left(\frac{\sigma_x}{\sigma_y}\right) y$ and $\gamma_{uv} = 0$, find the value of k.

18. (a) If $(A)=50$, $(B)=60$, $(C)=80$, $(AB)=35$, $(AC)=45$ and $(BC)=42$, find the greatest and least value of (ABC).

Or

(b) Show that for n attributes $A_1, A_2, \ldots, A_n$, $(A_1A_2\ldots A_n) \geq (A_1)+(A_2)+\ldots+(A_n)(n-1)N$.

19. (a) If x and y are two random variables, determine whether they are independent in the following cases.

(i) $f(x,y)=\begin{cases} 8xy; 0<x<1, 0<y<x \\ 0 \quad otherwise \end{cases}$

(ii) $f(x,y)=\begin{cases} 4xy; 0<x<1, 0<y<1 \\ 0 \quad otherwise \end{cases}$

Or

(b) State and prove the addition and multiplication theorems of expectation for continuous random variables.

20. (a) If X is a Poisson variate such that $P(X=2)=9\,P(X=4)+90P(X=6)$, find the value of β.

Or

(b) For a normal distribution, prove that $\mu_{2n}=(2n-1)\sigma^2\mu_{2n-2}$.

(7 pages)

Reg. No. : ....

Code No. : 20378 E **Sub. Code : CMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics – Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The radius of curvature for the curve $y = e^x$ at the point where it crosses the y-axis is ________.

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

2. If $x = f(t)$ and $y = g(t)$, then the radius of curvature is ________.

(a) $\frac{(x_1^2 + y_1^2)^{3/2}}{x_1 y_1 + x_2 y_1}$ (b) $\frac{(x_1^2 + y_1^2)^{3/2}}{x_2 y_2 + x_1 y_1}$

(c) $\frac{(x_1^2 - y_1^2)^{3/2}}{x_1 y_2 - x_2 y_1}$ (d) None of these

3. In Polar co-ordinates $\frac{\partial(x, y)}{\partial(r, \theta)} =$ ________.

(a) x (b) θ

(c) r (d) $r\theta$

4. $\int_0^1 \int_0^1 xy \, dx \, dy =$ ________.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) None of these

5. $\Gamma\left(\frac{1}{2}\right) =$ ________

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\pi}$

(c) π (d) $\frac{\sqrt{\pi}}{3}$

6. Transformations of Beta function $\beta\ (m,n)$ = ________

(a) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ (b) $\int_1^0 x^{m-1}(1-x)^{n-1}dx$

(c) $\int_0^\infty x^{m-1}(1-x)^{n-1}dx$ (d) None of these

7. The nth degree equation $f(x)=0$ can't have more than ________ roots.

(a) 4 (b) 6

(c) 7 (d) n

8. If α,β,γ are the roots of the equation $x^3-2x^2+5x-7=0$ then $\alpha\beta+\beta\gamma+\gamma\alpha$ = ________.

(a) 2 (b) 5

(c) –5 (d) 7

9. By removing the fractional co-efficients from $x^3-\frac{1}{4}x^2+\frac{1}{3}x-1=0$ multiply the roots by ________.

(a) 4 (b) 3

(c) –3 (d) 12

10. The equation $z^3-3z+1=0$ has a real root between ________.

(a) 1,2 (b) 2,3

(c) 3,4 (d) 4,5

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the radius of curvature of the cardioid $r=a(1-\cos\theta)$.

Or

(b) For the curve $x^3+y^3=3axy$, show that the radius of curvature is $\frac{3\sqrt{2}\,a}{16}$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$.

12. (a) If $x+y+z=u,\ y+z=uv,\ z=uvw$ prove $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$.

Or

(b) Evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$.

13. (a) Prove that $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta \, d\theta = \frac{1}{120}$.

Or

(b) Simplify $\int_0^{\frac{\pi}{2}} \sin^{10}\theta \, d\theta$.

14. (a) If α, β, γ are the roots of the equation $x^3 + px^2 + r = 0$ find the value of $\alpha^3 + \beta^3 + \gamma^3$.

Or

(b) Frame an equation with rational co-efficient, one of whose roots is $\sqrt{5} + \sqrt{2}$.

15. (a) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$

Or

(b) Transform the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$ whose roots are each diminished by 2.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find 'P' at the point 't' of the curve
$x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$

Or

(b) Find the equation of the evolute of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

17. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$.

Or

(b) Change the order of integration and evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy \, dx \, dy$.

18. (a) Show that $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$

Or

(b) Express $\int_0^1 x^m (1-x^n)^p \, dx$ interms of Gamma function and evaluate $\int_0^1 x^5 (1-x^3)^{10} \, dx$.

19. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in G.P.

Or

(b) Find correct to two place of decimals the root of the equation $x^4 - 3x + 1$ that lies between 1 and 2 by Newton method.

20. (a) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$

Or

(b) Discuss the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

(7 pages)

Reg. No. : ....

Code No. : 20379 E **Sub. Code : CMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics — Core

DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. Let m be the order of a given differential equation then

   (a) m is any integer

   (b) m is any real number

   (c) m is any positive integer

   (d) None of the above

2. What is the degree of the differential equation $3y = x\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right)$?

   (a) 1

   (b) 2

   (c) 8

   (d) None of the above

3. The roots of the auxiliary equation of the differential equation $(D^2 + 3D + 2)y = 0$ is ————

   (a) 1, 2

   (b) $-1, -2$

   (c) $-1, 2$

   (d) $1, -2$

4. The roots of the auxiliary equation of the differential equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$ is ————

   (a) 1, 1

   (b) $1 \pm i$

   (c) $-1, -1$

   (d) $1, -1$

5. If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two perpendicular lines then $a_1a_2 + b_1b_2 + c_1c_2 =$ ______

(a) 1

(b) −1

(c) 0

(d) None of the above

6. Angle between two diagonals of a cube is ______

(a) $\cos(1/3)$ (b) $\sin(1/3)$

(c) $\cos^{-1}(1/3)$ (d) $\sin^{-1}(1/3)$

7. If the line is parallel to the plane then $\sin\theta =$ ______

(a) 0

(b) 1

(c) $\frac{1}{\sqrt{2}}$

(d) None of the above

8. If the shortest distance in zero the lines are ______

(a) coplanar (b) non coplanar

(c) skewlines (d) none

9. Radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y - bz - 2 = 0$ is ______

(a) 2 (b) 3

(c) 4 (d) 5

10. The condition for orthogonality of two sphere is ______

(a) $2uu' + 2vv' + 2ww' = d + d'$

(b) $uu' + vv' + ww' = d + d'$

(c) $2uu' + 2vv' + 2ww' = d - d'$

(d) $uu' + vv' + ww' = d - d'$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve $xp^2 - 2yp + x = 0$

Or

(b) Solve $y = xp + x(1 + p^2)^{1/2}$

12. (a) Solve $(D^2 + 4)y = x \sin x$

Or

(b) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$

[P.T.O.]

13. (a) If the line whose direction cosines are given by $al + bm + cn = 0$ and $mn + nl + lm = 0$ are perpendicular, prove $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

Or

(b) Find the angle between $13x + 6y + 5z + 1 = 0$ and $6z - 4y - 2x + 81 = 0$

14. (a) Find the equation of the straight line through $(1, 0, 2)$ and parallel to the planes $2x + 3y - z = 1$ and $2x + y + z = 7$.

Or

(b) Find the angle between the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{6}$ and plane $3x + y + z = 1$.

15. (a) Find the equation of the sphere which passes through the points $(3, 4, 2)$, $(2, 0, 5)$, $(2, 4, 5)$, $(3, 3, 1)$.

Or

(b) Show that the plane $2x + y - 2z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x + 2y - 4z - 3 = 0$. find the point of contact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Solve $Z = px + qy + p^2q^2$

Or

(b) Solve $\frac{dx}{dt} + 2x - 3y = t$; $\frac{dy}{dt} - 3x + 2y = e^{2t}$

17. (a) Solve $(D^2 + 1)y = x^2e^{2x} + x\cos x$

Or

(b) Solve

$$(5 + 2x)^2 \frac{d^2y}{dx^2} - 6(5 + 2x)\frac{dy}{dx} + 8y = 6x$$

18. (a) Find the equation of the plane passing through the three points $(2, 3, 4)$, $(-3, 5, 1)$ and $(4, -1, 2)$.

Or

(b) Find the equation of the plane through the line of intersection of the planes $3x + 2y + 3 = 0$, $2x + y - z + 2 = 0$ and parallel to $x + y + z = 2$.

(8 pages) **Reg. No. :** ....

Code No. : 20380 E **Sub. Code : CMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics — Core

SEQUENCES AND SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The following statements are true except

(a) $\left(\frac{1}{n}\right)$ is a convergent sequence

(b) $\left(\frac{1}{n}\right)$ is a bounded sequence

(c) $\left(\frac{1}{n}\right)$ is a monotonic increasing sequence

(d) $\left(\frac{1}{n}\right)$ is a strictly mono

2. Read the following statements

(i) Any convergent sequence is a Cauchy sequence

(ii) Any Cauchy sequence is a convergent sequence

(iii) Any Cauchy sequence is a bounded sequence

(iv) Any bounded sequence is a Cauchy sequence

The correct statement

(a) only (i) and (iii) are true

(b) only (ii) and (iv) are true

(c) (i), (ii), (iii) and (iv) are true

(d) only (i) is true

3. The incorrect statement from the following (K_1, K_2)

(a) $1+2+3+4+\cdots$ diverges to ∞

(b) $\sum_1^\infty\left(\frac{1}{2^n}\right)$ converges to 1

(c) $\sum_1^\infty\left(\frac{1}{3^n}\right)$ converges to $\frac{1}{2}$

(d) $\sum_1^\infty\left(\frac{1}{n}\right)$ converges to 2

4. (i) The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p<1$

(ii) The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p>1$

The correct statement is ————

(a) only (i) is false

(b) only (ii) is false

(c) both (i) and (ii) are false

(d) both (i) and (ii) are true

5. $1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots=$ ————

(a) 2 (b) –2

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

6. $\lim_{n\to\infty}\frac{1}{n}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right)=$ ————

(a) 0 (b) e

(c) 1 (d) None

7. Let Σa_n be a series of positive terms. The correct statement from the following is

(a) Σa_n converges if $\lim_{n\to\infty}\frac{a_n}{a_{n+1}}>1$

(b) Σa_n converges if $\lim_{n\to\infty}\frac{a_n}{a_{n+1}}<1$

(c) Σa_n converges if $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}>1$

(d) Σa_n converges if $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}>0$

8. Applying the ratio test for $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}+\cdots$ the series is

(a) convergent

(b) divergent

(c) neither convergent nor divergent

(d) both convergent and divergent

9. $\lim_{n\to\infty}\left(1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}\right)=$ ————

(a) 0 (b) 1

(c) e (d) ∞

10. $\lim_{n\to\infty} \frac{(1^3+2^3+\cdots+n^3)}{n^4} =$ ________

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{4}$ (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Show that a sequence cannot converge to two different limits.

Or

(b) Prove that if Σa_n converges and Σb_n diverges then $\Sigma(a_n + b_n)$ diverges.

12. (a) If $(a_n) \to a$ and $(b_n) \to b$ prove that $(a_n b_n) \to ab$.

Or

(b) Test the convergence of the Geometric series $1 + r + r^2 + \cdots + r^n + \cdots$ when

(i) $0 \le r \le 1$

(ii) $r > 1$

(iii) $r = 1$.

13. (a) Discuss the convergence of the series $\sum \frac{1}{\sqrt{n^3+1}}$.

Or

(b) If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ prove that $x = \frac{y}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots$.

14. (a) Test the convergence of $\Sigma \frac{n^n}{n!}$.

Or

(b) Test the convergence of $\sum \sqrt{\frac{n}{n+1}} \cdot x^n$.

15. (a) Test the convergence of $\Sigma \frac{(-1)^n \sin n\alpha}{n^3}$.

Or

(b) State and prove Dirichlet's test.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Show that the sequence $\left(1+\frac{1}{n}\right)^n$ converges.

Or

(b) If $(a_n) \to a$ and $a_n \ne 0$ for all n and $a \ne 0$ then prove that $\left(\frac{1}{a_n}\right) \to \frac{1}{a}$. Also prove $\left(\frac{a_n}{b_n}\right) \to \frac{a}{b}$ if $(a_n) \to a, (b_n) \to b$ where $b_n \ne 0$ for all n and $b \ne 0$.

17. (a) Applying Cauchy's general principle of convergence prove that $1 - \frac{1}{2} + \frac{1}{3} - \cdots + (-1)^n \frac{1}{n} + \cdots$ is convergent.

Or

(b) Show that the harmonic series $\Sigma \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

18. (a) State and prove comparison test.

Or

(b) State and prove Kummer's test.

19. (a) Test the convergence of the series $1 + \frac{\alpha\beta}{r} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{r(r+1)2!} x^2 + \cdots$.

Or

(b) Test the convergence and divergence of the series $1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \frac{5^4 x^4}{5!} + \cdots$.

20. (a) State and prove Cauchy's condensation test.

Or

(b) Test the convergence of the series $\sum (-1)^n \left(\sqrt{n^2+1} - n\right)$.

———————

(6 pages)

Reg. No. : ....

Code No. : 20387 E **Sub. Code : CNMA 31**

U.G. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

Non Major Elective — MATHEMATICS FOR COMPETITIVE EXAMINATIONS — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $7+7\div 7\times 7 =$ ————

   (a) $\frac{2}{7}$ (b) 14

   (c) $7\frac{1}{7}$ (d) 42

2. $5005-5000\div 10.00 =$ ————

   (a) 0.5 (b) 50

   (c) 5000 (d) 4505

3. The average of first five multiples of 3 is

   (a) 9 (b) 72.6

   (c) 3 (d) 6

4. If $2A=3B=4C$ then $A:B:C$ is ————

   (a) 2 : 3 : 4 (b) 4 : 3 : 2

   (c) 6 : 4 : 3 (d) 3 : 4 : 6

5. ———— % of 64 is 8.

   (a) 3 (b) 10

   (c) 8 (d) 12.5

6. The number increased by $37\frac{1}{2}$% gives 33. The number is ————

   (a) 22 (b) 24

   (c) 25 (d) 27

7. A man sold a ratio for Rs. 1980 and gained 10% the ratio was bought for ————

   (a) Rs. 1782 (b) Rs. 1800

   (c) Rs. 2178 (d) Rs. 1500

8. Cost price = Rs. 56.25. profit = 20% selling price = ————

   (a) Rs. 62.50 (b) Rs. 60

   (c) Rs. 67.50 (d) Rs. 66.25

9. The difference of two numbers is 8 and $\frac{1}{8}$th of their sum is 35. The numbers are ————

   (a) 132, 140 (b) 128, 136

   (c) 124, 132 (d) 136, 144

10. 11 times a number gives 132. The number is ————

    (a) 11 (b) 12

    (c) 13.2 (d) 13

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find the value of $1+\cfrac{1}{2+\cfrac{1}{1-\cfrac{1}{3}}}$.

    Or

    (b) Find the value of $\frac{9^2\times 18^4}{3^{16}}$.

12. (a) The average age of a family of 6 members is 22 years. If the age of the youngest member be 7 years, find the average age of the family at the birth of the youngest member.

    Or

    (b) In a mixture of 35 litres, the ratio of milk and water is 4 : 1 now, 7 litres of water is added to the mixture. Find the ratio of milk and water in the new mixture.

13. (a) A person 'A' credits 15% of his salary in his fixed deposit account and spends 30% of the remaining amount on groceries. If the cash in hand is Rs. 2380, what is his salary?

    Or

    (b) A and B started a business and invested Rs. 20,000 and Rs. 25,000 respectively. After 4 months B left and C joined by investing Rs. 15,000. At the end of the year, there was a profit of Rs. 4,600. What is the share of C?

14. (a) By selling a watch for Rs. 144, a man loses 10%. At what price should he sell it to gain 10%?

    Or

    (b) A man sells an article at a profit of 20%. If he had bought it at 20% less and sold if for Rs. 5 less, he would have gained 25%. Find the cost price of the article.

15. (a) A fraction becomes 4 when 1 is added to both numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. What is the numerator of the fraction?

Or

(b) The number x is exactly divisible by 5 and the remainder obtained on dividing the number y by 5 is 1. What remainder will be obtained when $x+y$ is divided by 5?

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Evaluate $\frac{0.125+0.027}{0.25-0.15+0.09}$.

Or

(b) The average of 5 consecutive numbers is n. If the next two numbers are also included, the average will be increased by how much?

17. (a) Two numbers are in the ratio 3 : 5. If each number is increased by 10, the ratio becomes 5 : 7. Find the numbers.

Or

(b) The ratio of milk and water in 85 Kg of adulterated milk is 27 : 7. Find the amount of water which must be added to make the ratio 3 : 1.

18. (a) A and B invest in the business in the ratio 3 : 2. If 5% of the total profit goes to charity and A's share is Rs. 855 then find the total profit?

Or

(b) 72% of the students of a certain class took biology and 44% took Mathematics. If each student took biology or mathematics and 40 took both, find the total number of students in the class.

19. (a) 'A' bought 25 kg of rice at rate of Rs. 6 per kg and 35 kg of rice at the rate of Rs. 7 per kg. He mixed the two and sold the mixture at the rate of Rs. 6.75 per kg. What was his profit or loss in the transaction?

Or

(b) A bought a TV with 20% discount on the labelled price. Had he bought it with 25% discount he would have saved Rs. 500. At what price did he buy the TV?

20. (a) The sum of squares of two numbers is 80. And the square of their difference is 36. Find the product of the numbers.

Or

(b) Of the three numbers, the sum of first two is 45; the sum of the second and the third is 55 and the third and thrice the first is 90. What is the third number?

(8 pages) **Reg. No. :** ....

Code No. : 20386 E **Sub. Code : CSMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

Skill Based Subject — VECTOR CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The directional derivative of $\phi(x,y,z) = x^3 + y^3 + z^3$ at the point (1, –1, 2) is ————

 (a) $3\bar{i} + 4\bar{j} + 3\bar{k}$ (b) $3\bar{i} + 3\bar{j} + 12\bar{k}$

 (c) $3\bar{i} + 3\bar{j} + 3\bar{k}$ (d) $3\bar{i} + 2\bar{j} + 2\bar{k}$

2. The unit vector normal to the surface $\phi = C$ is ————

 (a) $\frac{\nabla\phi}{|\nabla\phi|}$ (b) $\nabla\phi$

 (c) $\nabla^2\phi$ (d) $\frac{|\nabla\phi|}{\nabla\phi}$

3. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla.\bar{r} =$ ————

 (a) $2x$ (b) $3y$

 (c) 3 (d) 4

4. If the vector $(2x,z)\bar{i} + (4x - 11y + 3z)\bar{j} + (3x + mz)\bar{k}$ is solenoidal, then the value of m is ————

 (a) 3 (b) 9

 (c) 2 (d) 11

5. If $\bar{f} = x^2\bar{i} - xy\bar{j}$ and C is the straight line joining the points (0, 0) and (1, 1), then $\int_C \bar{f}.d\bar{r}$ is ————

 (a) 1 (b) 0

 (c) –1 (d) 2

6. If $\overline{F} = z(x\overline{i} + y\overline{j} + z\overline{k})$ and C is the straight line joint (0, 0, 0) and (1, 1, 1), then $\int_C \overline{f}.d\overline{r}$ is ————

(a) 0 (b) −1

(c) 1 (d) 2

7. If S is any closed surface enclosing a volume V and $\overline{f} = ax\overline{i} + by\overline{j} + cz\overline{k}$, then $\iint_S \overline{f}.\overline{n}\,dS =$ ————

(a) $(a+b+c)V$ (b) $3V$

(c) $(a+b+c)^3V^3$ (d) 0

8. The value of $\int_0^a\int_0^a\int_0^a x^2ydzdydx$ is ————

(a) $\frac{a^3}{3}$ (b) $\frac{a^4}{5}$

(c) $\frac{a^5}{4}$ (d) $\frac{a^6}{6}$

9. The value of $\int_C (3x+4y)dx + (2x-3y)dy$, where C is the circle $x^2+y^2=4$ is ————

(a) 4π (b) -8π

(c) 8π (d) 2π

10. The value of $\iint_C \left[(1+y)z\overline{i} + (1+z)x\overline{j} + (1+x)y\overline{k}\right].d\overline{r}$, where C is a closed curve in the plane $x-2y+z=1$ is ————

(a) 2 (b) −1

(c) 0 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) Find the directional derivative of $\phi = x + xy^2 + yz^3$ at (0, 1, 1) in the direction of the vector $2\overline{i} + 2\overline{j} - \overline{k}$.

Or

(b) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $|\overline{r}| = r$, prove that $\nabla\left(\frac{1}{r}\right) = \frac{-\overline{r}}{r^3}$.

12. (a) If $\overline{A} = axy\overline{i} + (x^2+2yz)\overline{j} + y^2\overline{k}$ is irrotational, find the value of 'a'.

Or

(b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where 'n' is a constant.

13. (a) Evaluate $\int_C \overline{F}.d\overline{r}$ where $\overline{F} = yz\overline{i} + xz\overline{j} - xy\overline{k}$ and C is the straight line having end points $O(0,0,0)$ and $P(2,4,8)$.

Or

(b) If $\overline{f} = 3xy\overline{i} - y^3\overline{j}$, compute $\int_C \overline{F}.d\overline{r}$ along $y = 2x^2$ from (0, 0) to (1, 2).

14. (a) If $\overline{A} = curl\overline{F}$, compute $\iint_S \overline{A}.\hat{n}\, dS$ for any closed surface S.

Or

(b) Evaluate $\iiint_V \nabla.\overline{F}\, dV$ if $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ and if V is the volume of the region enclosed by the cube $0 \le x,y,z \le 1$.

15. (a) Evaluate $\int_C xydx - x^2dy$ by converting it into a double integral. If is given that the boundary of the region bounded by the line $y = x$ and the parabola $x^2 = y$.

Or

(b) Evaluate by Green's theorem $\int_C e^{-x}(\sin ydx + \cos ydy)$ where C is the rectangle with vertices $(0,0),(\pi,0),(\pi,\pi/2),(0,\pi/2)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) If $\nabla\phi = (y + y^2 + z^2)\overline{i} + (x + z + 2xy)\overline{j} + (y + 2zx)\overline{k}$ and if $\phi(1,,11) = 3$, find ϕ.

Or

(b) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $\overline{r} = r\hat{r}$ show that

(i) $\nabla\, (f(r)\overline{r} = rf'(r) + 3f(r)$ (ii) $\nabla \times (f(r)\overline{r}) = \overline{0}$.

17. (a) Find the value of 'm' if $\overline{F} = (6xy + z^3)\overline{i} + (mx^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$ is irrotational. Find also ϕ such that $\overline{F} = \nabla\phi$.

Or

(b) Show that

(i) $(\overline{V}.\nabla)\overline{r} = \overline{V}$

(ii) $(\overline{V} \times \nabla) \times \overline{r} = -2\overline{V}$.

18. (a) If $\overline{f} = (3x^2 + 6y)\overline{i} - 14yz\overline{j} + 20xz^2\overline{k}$, evaluate $\int_C \overline{F}.d\overline{r}$ where C is (i) a curve whose parametric equations are $x = t, y = t^2, z = t^3$ (ii) straight lines OA, AB, BP where A is $(1, 0, 0)$, B is $(1, 1, 0)$, $O(0, 0, 0)$ and P is $(1,1,1)$.

Or

(b) Evaluate $\iint_S \overline{A}.\overline{n}\,dS$ over the surface S of the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$ if $\overline{A} = 4x\overline{i} - 2y^2\overline{j} + z^2\overline{k}$.

19. (a) Verify Gauss divergence theorem for $\overline{f} = (x^3 - yz)\overline{i} - 2x^2y\overline{j} + 2\overline{k}$ over the cube bounded by $x = 0, y = 0, z = 0$, $x = a, y = a, z = a$.

Or

(b) Verify Gauss divergence theorem for $\overline{A} = a(x + y)\overline{i} + a(y - x)\overline{j} + z^2\overline{k}$ taken over the region V bounded by the upper hemisphere $x^2 + y^2 + z^2 = a^2$ and the plane $z = 0$.

20. (a) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region R enclosed by the parabolas $y = x^2$ and $y^2 = x$.

Or

(b) Verify Stoke's theorem for $\overline{F} = y^2\overline{i} + y\overline{j} - xz\overline{k}$ over the upper half of the surface of the sphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

Reg. No. : ....

Code No. : 20075 E **Sub. Code : SAST 21/ AAST 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second/Fourth Semester

Mathematics — Allied

STATISTICS — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The ———— mean of Laspeyre's and Paasche's index number is the Bowley's index number.

   (a) arithmetic

   (b) geometric

   (c) harmonic

   (d) none of the above

2. The ———— year is the period against which comprarision is made.

(a) base

(b) current

(c) upcoming

(d) none of the above

3. The standard error of $\bar{x}_1 - \bar{x}_2$ is ————

(a) $\sqrt{\frac{\sigma^2}{2n}}$ (b) $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n^2}}$

(c) $\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^3}{2n_2}}$ (d) none of the above

4. The standard error of sample variance s^2 is ————

(a) $\frac{\sigma}{\sqrt{n}}$ (b) $\sqrt{\frac{\sigma^2}{2n}}$

(c) $\sigma^2\sqrt{\frac{2}{n}}$ (d) none of the above

5. F test is always ————

(a) two tailed test

(b) right - tailed test

(c) left tailed test

(d) none of the above

6. For the 2×2 contingency table

83	57
45	68

, the value of x^2 =

(a) 10

(b) 9

(c) 9.48

(d) none of the above

7. Analysis of variance (ANOVA) is developed by

(a) Bowley

(b) Kelley

(c) Fisher

(d) None of the above

8. The total degree of freedom for a random sample of N values is

(a) $\frac{N}{2}$ (b) $2N$

(c) $N-1$ (d) None of the above

9. Control chart is developed by

(a) Shewalt

(b) Taylor

(c) Euler

(d) None of the above

10. The number of horizontal lines in the control chart is ————

(a) 4 (b) 2

(c) 3 (d) none of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) Calculate by the weight Aggregate method, the index number from the following data.

Commodity	Base Year (Price per unit)	Current year Price per unit	Weight
Rice	30	40	10
Wheat	20	30	5
Pulses	40	50	6
Oil	35	40	5
Milk	40	50	10

Or

(b) Calculate peice index number for 1945 by (i) Bowley's method and (ii) Fisher's method.

	1935		1945	
Commodity	Price in Rs.	Quantity	Price in Rs.	Quantity
A	4	50	10	40
B	3	10	9	2
C	2	5	4	2

12. (a) A salesman in a departmental store claims that at most 60% of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Is the sample result consistent with the claim of the salesman? Use a level of significance of 5%.

Or

(b) In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the sample have been drawn from the sample population with S.D. 4?

13. (a) The mean height and the S.D. height of 8 randomly chosen boy students are 166.9 cm and 8 29 cm respectively. The corresponding values of 6 randomly chosen girl students are 170.3 cm and 8.50 cm respectively. Based on this data, can we conclude that boy students are, in general, shorter than girl students. (t value for 12 d.f. = 1.782)

Or

(b) A certain injection administered to each of 12 patients resulted in the following change of blood pressure : 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the injection will, in general, result in increase of B.P? (t value = 1.80 for 11 d.f)

14. (a) Explain basic designs of experiment.

Or

(b) Four salesmen were posted in different areas by a company. The number of units of commodity X sold by them are as follows. On the basis of this information can it be concluded that there is a significant difference in the performance of the salesmen. $F_{(3,12)}$ $d.f = 3.24$

A	20	23	28	29
B	25	32	30	21
C	23	28	35	18
D	15	21	19	25

15. (a) The average number of defectives in 22 sample lots of 2,000 rubber belts was found to be 16%. Obtain the values for central line and control limits for p-chart.

Or

(b) Draw mean chart for the following 10 samples mean of size 5 each 43, 49, 37, 44, 45, 37, 51, 46, 43 and 47. Comment on the state of control of the process. ($n = 5$, $A_2 = 0.58$)

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Construct (i) Chain Base Index (ii) Fixed Base Index taking 1993 as origin :

Year	Price (in Rs.) per quintal
1963	50
1964	60
1965	62
1966	65
1967	70
1968	78
1969	82
1970	84
1971	88
1972	90

Or

(b) The table below gives the princes of base year and current year of 5 commodities with their quantities. Use it to verify whether Fishere's index satisfies factor reversal test and time reversal test.

Commodity	Price year		Current year	
	Unit Price (Rs.)	Quantity	Unit Price (Rs.)	Quantity
A	5	50	5	70
B	5	75	10	80
C	10	80	12	100
D	5	20	8	100
E	10	50	5	60

17. (a) On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups, the upper 30 percent and the remaining 70 percent. Consider the first question of this examination..Among the first group, 40 had the correct answer, whereas among the second group. 80 had the correct answer. On the basis of these results, can we conclude that the first question is no good at discriminating ability of the type being examined here.

Or

(b) Random samples drawn from two countries gave the following data relating to the heights of adult males.

	Country A	Country B
Mean height in inches	67.42	67.25
Standard deviation	2.58	2.50
Number in samples	1000	1200

(i) Is the difference between the means significant?

(ii) Is the difference between the standard deviations significant.

18. (a) Fit a poisson distribution to the following data and test the goodness of fit. Also given χ^2 for 2 d.f at 5% level of significance is 5.99.

x	0	1	2	3	4	5	6
f	275	72	30	7	5	2	1

Or

(b) A random sample of 10 boys had the following IQ's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q of 100 . Find a reasonable range in which most of the mean IQ values of samples of 10 boys lie.

19. (a) Five types of treatments are given. The number average and standard deviation for each treatment are given in the following table. Test whether the treatments are homogeneous.

	A	B	C	D	E
Treatment No :	10	6	8	11	5
Mean :	10.9	13.5	11.5	11.2	15.4
Standard deviation :	12.72	5.96	3.24	5.65	3.64

Or

(b) Carry out analysis of variance for data of 7 varities, 5 observations being taken on each variety.

Variety No :	1	2	3	4	5	6	7
Observation No :							
1	13	15	14	14	17	15	16
2	11	11	10	10	15	9	12
3	10	13	12	15	14	13	13
4	16	18	13	17	19	14	15
5	12	12	11	10	12	10	12

20. (a) The following table gives the number of defective items found in 20 successive samples of 100 items each.

2	6	2	4	4	15	0	4	10	18
2	4	6	4	8	0	2	2	4	0

Comment whether the process is under control suggest suitable control limits for the future.

Or

(b) Ten pieces of cloth contained the following number of defects : 3, 0, 2, 8, 4, 2, 1, 3, 7, 1. Prepare a C-chart and state whether the production process is in a state of control.

ages)

Reg. No. : ....

.de No. : 30352 E **Sub. Code : SAST 21/ AAST 21**

:BCS) DEGREE EXAMINATION, APRIL 2022.

Second/Fourth Semester

Mathematics – Allied

STATISTICS – II

'or those who joined in July 2017 onwards)

Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

hoose the correct answer :

isher's index number is ——— of Laspeyer's nd Paasche's index number.

ı) arithmetic mean (b) geometric mean

:) harmonic mean (d) none

'he factor reversal test is $I_{pq} \times I_{qp} =$ ———

a) $\frac{\sum p_1 q_0}{\sum p_0 q_1}$ (b) $\frac{\sum p_1 q_0}{\sum p_1 q_1}$

c) $\frac{\sum p_1 q_1}{\sum p_0 q_1}$ (d) $\frac{\sum p_1 q_1}{\sum p_0 q_0}$

The standard deviation of the sampling listribution of a statistic is known as ———

(a) normal error (b) standard error

(c) type I error (d) type II error

4. The sample is said to be large if its sample size exceeds ———

(a) 100 (b) 50

(c) 40 (d) 30

5. t-distribution was done by ———

(a) W.S. Gosset (b) Karl Pearson

(c) R.A. Fisher (d) Royden

6. The value of χ^2 range from ———

(a) $-\infty$ to ∞ (b) 0 to ∞

(c) -1 to 1 (d) 0 to 1

7. If k denotes number of rows and h denotes number of columns then the mean square value between the rows in two criteria of classification is ———

(a) $\frac{v_1}{(k-1)}$ (b) $\frac{v_2}{(h-1)}$

(c) $\frac{v_1}{(h-k)}$ (d) $\frac{v_2}{(h-k)}$

8. In three criteria of classification the degrees of freedom between the rows is ———

(a) n (b) $n-1$

(c) $(n-2)$ (d) $(n-1)(n-2)$

9. S.Q.C techniques were developed by ———.

(a) W.A. Shewhart (b) A.L. Bowley

(c) Karl Pearson (d) Edgeworth

10. The Upper Control Limit for R chart is ———.

(a) $D_1\overline{R}$ (b) $D_2\overline{R}$

(c) $D_3\overline{R}$ (d) $D_4\overline{R}$

PART B — (5 × 5 = 25 marks)

nswer ALL questions, choosing either (a) or (b).

(a) Find the value of x in the following data if the ratio between Laspeyre's and Paasche's index number is 28:27.

Commodities	p_0	q_0	p_1	q_1
A	1	10	2	5
B	1	5	x	2

Or

(b) From the following data construct an index number for 1970 taking 1969 as the base by price relatives method using (i) A.M (ii) G.M for averaging the relatives.

Commodities	Price in 1969 Rs.	Price in 1970 Rs.
A	150	170
B	40	60
C	80	90
D	100	120
E	20	25

(a) A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39,350 kms, with a standard deviation of 3,260. Could the sample come from a population with mean life of 40,000 kms? Establish 99% confidence limits within which the mean life of tyres is expected to lie.

Or

(b) Intelligence test on two groups of boys and girls gave the following results :

	Mean	S.D	N
Girls	75	15	150
Boys	70	20	250

Is there a significant difference in the mean scores obtained by boys and girls?

13. (a) A random sample of size 16 has 53 as mean. The sum of the squares of the deviations taken from mean is 135. Can this sample be regarded as taken from the population having 56 as mean?

Or

(b) The mean life of a sample of 10 electric light bulbs was found to be 1456 hours with S.d of 423 hours. A second sample of 17 bulbs chosen from a different batch showed a mean life of 1280 hours with S.d of 398 hours. Is there a significant difference between the means of the two samples?

14. (a) The yields of 3 varieties of wheat in 3 blocks are given below. Is the difference between the varieties significant?

Variety \ Block	1	2	3
A	10	9	8
B	7	7	7
C	8	5	4

Or

(b) Write a short note on two criteria of classification.

[P.T.O.]

15. (a) The following table gives the inspection data on completed spark plugs.

(2000 Spark plugs in 20 lots of 100 each)

Lot Number	Number Defectives	Fraction Defectives
1	5	0.50
2	10	0.100
3	12	0.120
4	8	0.080
5	6	0.060
6	5	0.050
7	6	0.060
8	3	0.030
9	3	0.030
10	5	0.050
11	4	0.040
12	7	0.070
13	8	0.080
14	2	0.020
15	3	0.030
16	4	0.040
17	5	0.050
18	8	0.080
19	6	0.060
20	10	0.100

Construct p-chart.

Or

(b) Explain – Acceptance sampling.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Calculate :

(i) Laspeyre's (ii) Paasche's

(iii) Bowley's (iv) Fisher's

(v) Marshall's Edgeworth's

Index numbers for the following data given below

Commodity	Base Year		Curent year	
	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	15

Or

(b) Construct with a help of data given below. Fisher's index number and show that it satisfies both the factor reversal test and time reversal test

Commodity	A	B	C	D
Base year price in Rupees	5	6	4	3
Base year quantity in Quintals	50	40	120	30
Current year in Rupees	7	8	5	4
Current year quantity in Quintals	60	50	110	35

17. (a) A dice is thrown 9000 times and a throw of 3 or 4 observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies?

Or

(b) In a random sample of 1,000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveal significant difference between town A and town B, so far, as the proportion of wheat consumers is concerned?

18. (a) Two random samples were drawn from two normal populations and their values are

A: 66 67 75 76 82 84 88 90 92

B: 64 66 74 78 82 85 87 92 93 95 97

Test whether the two populations have the same variance at the 5% level of the significance.

Or

(b) In an experiment on pea-breeding Mendel obtained the following frequencies of seeds : 315 round and yellow, 101 wrinkled and yellow, 108 round and green, 32 wrinkled and green. According to his theory of heredity, the numbers should be in proportion 9:3:3:1. Is there any evidence to doubt the theory at 5% level of significance?

19. (a) Analyse the variance in the following Latin square.

B20	C17	D25	A34
A23	D21	C15	B24
D24	A26	B21	C19
C26	B23	A27	D22

Or

(b) There varieties of cows of same age group are treated with four different types of fodders. The yields milk in deciliters are given below. Perform an analysis of variance and check whether is any significant difference between the yields of different varieties of cows due to different types of fodders.

Varieties of cows \ Fodder	f_1	f_2	f_3	f_4
C_1	61	63	66	68
C_2	62	64	67	69
C_3	63	63	68	69

20. (a) Construct $\overline{X}$ and R charts for the following data.

Sample number	Observations		
1	32	37	42
2	28	32	40
3	39	52	28
4	50	42	31
5	42	45	34
6	50	29	21
7	44	52	35
8	22	35	44

Or

(b) Explain control charts and its types.

(6 pages) Reg. No. : ....

Code No. : 20083 E **Sub. Code : SEMA 5 B/ AEMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics

Major Elective — DISCRETE MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL the questions.

Choose the correct answer

1. The statement Q is called the ———— in $P \to Q$

   (a) antecedent (b) consequent

   (c) tautologies (d) None

2. If P and Q are two statements, then the statement $P \to Q$ is called ————

   (a) Conditional statement

   (b) Biconditional statement

   (c) Simple statement

   (d) None

3. Every axiom is a ————

   (a) lemma (b) statement

   (c) theorem (d) formula

4. A sum of the variables and their negations is called on ————.

   (a) elementary sum (b) elementary product

   (c) normal sum (d) none

5. A group $< G,* >$ in which the operation $*$ is commutative is called an ———— group.

   (a) semi (b) subgroup

   (c) abelian (d) none

6. Semigroup $< M,0 >$ with an identity element with respect to the operation 0 is called a ————.

   (a) monoid (b) group

   (c) abelian (d) permutation

7. Every ———— is a distributive lattice.

   (a) chain (b) group

   (c) bounds (d) homomorphism

8. A ———— algebra is a complemented, distributive lattice.

   (a) Boolean (b) Partial

   (c) Ordinary (d) None

9. What are the numbers using for represent octal number?

   (a) 0-9 (b) 0-1

   (c) 0-7 (d) none

10. Subtract 01110 from 10101?

    (a) 11001 (b) 100100

    (c) 00111 (d) 11001

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Construct the truth table for Biconditional.

    Or

    (b) Construct the truth table for $(P \vee Q) \vee \urcorner P$

12. (a) Symbolize the statement: "All men are giants".

    Or

    (b) Prove that $(fx)(P(x) \wedge Q(x)) \Rightarrow (fx)P(x) \wedge (fx)Q(x)$.

13. (a) Show that the kernel of a homomorphism g from a group $< G,* >$ to $< H,\Delta >$ is a subgroup of $< G,* >$.

    Or

    (b) Prove that the parity – check matrix H generates a code word of weight q if there exists a set of q columns of H such that their k-tuple sum is zero.

14. (a) Let $< L,*,\oplus >$ be a distributive lattices. Then prove that for any $a,b,c,\in L, (a*b = a*c)$

    $\wedge (a \oplus b = a \oplus c) \Rightarrow b = c$

    Or

    (b) Prove the Boolean identify $(A+B)(A+C) = A + BC$.

15. (a) Convert $(101010101)_2$ to hexadecimal.

    Or

    (b) Divide 100001 by 110?

[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Construct the truth table for $(Q \wedge (P \to Q)) \to P$.

Or

(b) Does the formula $(\neg Q \wedge P) \wedge Q$ is tautology or not.

17. (a) Obtain the principal disjunctive norm forms of

(i) $\neg P \vee Q$,

(ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Or

(b) Does P follows from $(P \vee Q)$.

18. (a) Write a definition of Group and subgroup.

Or

(b) Prove that the minimum weight of the nonzero code words in a group code is equal to minimum distance.

19. (a) If any Boolean algebra, show that $a = b \Leftrightarrow ab' + a'b = 0$.

Or

(b) When $< B, *, \oplus >$ becomes a lattice.

20. (a) Convert the following to octal numbers (i) 110101110_2 (ii) 111101.01101_2

Or

(b) Multiply :

(i) 1.01×10.1

(ii) 100101×1001

(8 pages)

Reg. No. : ....

Code No. : 20085 E Sub. Code : SEMA 5 D/ AEMA 54

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics

Major Elective — OPERATIONS RESEARCH – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Linear programming problem involving only two decision variables can be solved by
   (a) Graphical method
   (b) Simplex method
   (c) Both (a) and (b)
   (d) None of the above

2. Simplex method was introduced by
   (a) G.B Dantzig (b) Konig
   (c) Miller (d) Taha

3. If the primal problem has an unbounded solution then the dual problem has ————
   (a) feasible solution
   (b) basic solution
   (c) no feasible solution
   (d) optimal solution

4. If the dual problem has n variables, then the primal problem has ———— constraints.
   (a) n (b) $n+1$
   (c) $n-1$ (d) none

5. All the basis for a transportation problem are
   (a) triangular (b) non-triangular
   (c) equal (d) unequal

6. Which of the following method used to find the optimum solution of transportation problem?
   (a) VAM method
   (b) North-West corner rule
   (c) MODI method
   (d) Matrixminima method

7. In a balanced assignment problem the cost matrix is
   (a) Symmetric matrix
   (b) Square matrix
   (c) Unsymmetric matrix
   (d) Not a square matrix

8. If an assignment problem having 4 workers and 3 jobs, then the total number of possible assignment is
   (a) 4 (b) 3
   (c) 7 (d) 12

9. If indicates the time required by a job on each machine
   (a) Elapsed time (b) Processing time
   (c) Idle time (d) None

10. Number of sequences require to evaluate sequencing problem with 6 jobs, 5 machines
    (a) $(6!)^5$ (b) $(5!)^6$
    (c) $5!\times 6$ (d) 5×6

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve the following LPP using graphical method

Maximize $z = x_1 + x_2$

S.T

$x_1 + x_2 \le 1$
$-3x_1 + x_2 \ge 3$
$x_1, x_2 \ge 0.$

Or

(b) Solve the following LPP, using graphical method

Minimize $z = x_1 + x_2$

S.t

$5x_1 + 3x_2 \le 15$
$x_1 + x_2 \ge 6$
$x_1, x_2 \ge 0.$

12. (a) Write the dual of the following LPP.

Minimize $z = 4x_1 + 6x_2 + 18x3$

S.t

$x_1 + 3x_2 \geq 3$

$x_2 + 2x_3 \geq 5$

$x_1, x_2, x_3 \geq 0.$

Or

(b) Prove that the dual of the dual is primal.

13. (a) Explain the North - West corner rule.

Or

(b) Obtain an initial basic feasible solution to the following transportation problem.

	D	E	F	G	
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
	200	225	275	250	

14. (a) Write the mathematical formulation of the assignment problem.

Or

(b) State and prove reduction theorem.

15. (a) Explain the basic terms used in sequencing problem.

Or

(b) Find the optimum sequence of the following data.

Jobs :	J_1	J_2	J_3	J_4	J_5	J_6
Machine A :	1	3	8	5	6	3
Machine B :	5	6	3	2	2	10

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Use penalty method to solve the LPP.

Maximize $z = 6x_1 + 4x_2$

S.t

$2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0.$

Or

(b) Explain the simplex algorithm.

17. (a) Use duality to solve

Maximize $z = 2x_1 + x_2$

S.t.

$x_1 + x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0.$

Or

(b) Explain the dual simplex method.

18. (a) Write the transportation algorithm in detail.

Or

(b) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
	3	3	2	2	

19. (a) Write the Hungarian algorithm.

Or

(b) Solve the following assignment problem.

	1	2	3
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

20. (a) Solve the following sequencing problem.

Jobs :	A	B	C	D	E	F	G	H	I
M_1:	2	5	4	9	6	3	7	5	4
M_2:	6	8	7	4	3	9	3	8	11

Or

(b) Explain the processing 2 jobs through K machines.

) A company operating 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs. 240 a metre and there is a demand for 8,000 meteres a week. Each replenishment costs Rs. 1,050 for administration and Rs. 1,650 for delivery, while holding costs are estimated at 25 percent of value held a year. Assume no shortages are allowed, what is the optimal inventory policy for the company? How would this analysis differ if the company wanted to maximize profit rather than minimize cost? What is the gross profit if the company sell cable for Rs. 360 a metre.

Or

The demand for an item in a company is 18,000 units per year, and the company can produce the items at a rate of 3,000 per month. The costs of one set-up is Rs. 500 and holding cost of 1 unit per month is 15 paise. The storage cost of one unit is Rs. 200 per month. Determine (i) optimum production batch quantity and the number of strategies, (ii) optimum cycle time and the production time, (iii) maximum inventory level in the cycle and (iv) total associated cost per year of the cost of the time is Rs. 20 per unit.

Reg. No. : ....

Code No. : 30368 E **Sub. Code : SEMA 6 D**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics

Major Elective — OPERATIONS RESEARCH - II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A game is said be fair, if ————

(a) upper value is more than lower value of the game

(b) upper and lower values are not equal

(c) upper and lower values are same and zero

(d) upper value is less than lower value of the game

2. The size of the pay-off matrix of a game can be reduced by using the principle of ——————

(a) dominance (b) rotation reduction

(c) game inversion (d) game transpose

3. The problem of replacement is not concerned about the ——————

(a) items are deteriorate graphically

(b) determination of optimum replacement interval

(c) items that fails suddenly

(d) maintenance of an item to work out profitability

4. Mortality problems ——————

(a) are special type of problems, where failure is treated as birth and the replacement of an item on the failure is treated as death

(b) uses mortality tables to derive the probability distribution of the life span of an equipment/item

(c) are like replacement policies for items whose value does not deteriorate gradually

(d) none of the above

(i) Find the critical path and the expected time of the project.

(ii) Find the total and free-float for each activity.

Or

(b) A project is composed of eight activities, the time estimates for which are given below.

Activity		Time required (days)		
Event	Name	t_o	t_m	t_p
1-2	A	6	6	24
1-3	B	6	12	18
1-4	C	12	12	30
2-5	D	6	6	6
3-5	E	12	30	48
4-6	F	12	30	42
5-6	G	18	30	54

(i) Find the expected duration and variance of each activity.

(ii) What is the expected project length?

(iii) Calculate the variance and standard deviation of the project length.

If the tubes are group replaced, the cost of replacement is Rs. 15 per tube. Group replacement can be done at fixed intervals at fixed intervals in the night shift when the computer is not normally used. Replacement of individual tubes which fail in services costs Rs. 60 per tube. How frequently should the tubes be replaced?

a) Explain the solution procedure of the queueing model (M/M/1) : (∞/FIFO). Also obtain its characteristics.

Or

b) Explain the queueing model (M/M/1) : (N/FIFO) and find its characteristics.

a) The following are the details of estimated times of activities of a certain project.

Activity	Immediate predecessors	Normal time (days)
A	–	16
B	–	20
C	A	8
D	A	10
E	B,C	6
F	D,E	12

5. Queue can form only when ————

(a) arrivals exceed service capacity

(b) arrivals equals service capacity

(c) service facility is capable to serve all the arrivals at a time

(d) there are more than one service facilities

6. Priority queue discipline may be classified as ————

(a) finite or infinite

(b) limited or unlimited

(c) pre-emptive or non-pre-emptive

(d) all of the above

7. Network problems have advantage in terms of project ————

(a) scheduling (b) planning

(c) controlling (d) all of the above

8. In critical path analysis, CPM is ————

(a) event oriented

(b) probabilistic in nature

(c) deterministic in nature

(d) dynamic in nature

9. Which of the following is not an assumption underlying the fundamental problem of EOQ?

(a) demand is known and uniform

(b) lead time is not zero

(c) holding cost per unit time period is constant

(d) stock-outs are not permitted

10. If the procurement cost used in the formula to compute EOQ is half of the actual procurement cost, the EOQ so obtained will be ————

(a) half of EOQ (b) 0.707 time EOQ

(c) one third of EOQ (d) one forth of EOQ

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Solve the game whose payoff matrix is given by

Player B

Player A	B_1	B_2	B_3
A_1	1	3	1
A_2	0	–4	–3
A_3	1	5	–1

Or

17. (a) A manufacturer is offered two machines A and B. A is priced at Rs. 5,000 and running costs are estimated at Rs. 800 for each of the first years, increasing by Rs. 200 per year in the sixth and subsequent years. Machine B which has the same capacity as A costs Rs. 2,500 but will have running costs of Rs. 1,200 per year for six years, increasing by Rs. 200 per year thereafter. If money is worth 10% per year, which machine should be purchased? (Assume that the machine will eventually be sold for scrap at a negligible price)

Or

(b) A computer has a large number of electronic tubes. They are subject to mortality as given below.

Period	Age of failure (hours)	Probability of failure
1	0-200	0.10
2	201-400	0.26
3	401-600	0.35
4	601-800	0.22
5	801-1000	0.07

) The demand for a certain items is 16 units per period. Unsatisfied demand causes a shortage cost of Re. 0.75 per unit per short period. The cost of initializing purchasing action is Rs. 15.00 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8.00 per unit. (Assume that shortages are being back ordered at the above mentioned cost). Find the minimum cost purchase quantity.

PART C — (5 × 8 = 40 marks)

swer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

ı) Use graphical method in solving the following game :

Player A

$$\text{Player B}\begin{pmatrix} 2 & 2 & 3 & -2 \\ 4 & 3 & 2 & 6 \end{pmatrix}$$

Or

) Solve the following game by using dominance property.

Player A

$$\text{Player B}\quad \begin{matrix} I \\ II \\ III \\ IV \end{matrix}\begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

(b) For the game with the following payoff matrix, determine the optimum strategies and the value of the game :

$$\begin{matrix} & P_2 \\ P_1 & \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix} \end{matrix}$$

12. (a) A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value, Rs. 200.

The running (maintenance and operating) cost in rupees are found from experience to be as follows :

Years :	1	2	3	4
Running cost :	200	500	800	1,200
Years :	5	6	7	8
Running cost :	1,800	2,500	3,200	4,000

When should the machine be replaced?

Or

(b) The cost of new machine is Rs. 15,000. The maintenance cost of n^{th} year is given by

$C_n = 500(n-1)$; n = 1, 2,.. suppose that the discount rate per year is 0.5. After how many years if will be economical to replace the machine by a new one?

13. (a) A T.V. repairman finds that the time spent on his jobs has an exponential distribution with 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is approximately.

Poisson with an average rate of 10 per 8 hour day. What is repairman's expected idle time each day? How much jobs are ahead of the average set just brought in?

Or

(b) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard.

14. (a) Given the following information :

Activity :	0-1	1-2	1-3	2-4	2-5
Duration (in days) :	2	8	10	6	3
Activity :	3-4	3-6	4-7	5-7	6-7
Duration (in days) :	3	7	5	2	8

(i) Draw the arrow diagram.

(ii) Identify critical path and find the project duration.

Or

(b) Consider the data of the project, find its critical path and project duration.

Activity	A	B	C	D	E	F	G	H	I
Predecessor	–	–	A	B	C,D	B	E	E	F,G
Duration (days)	4	7	2	9	6	5	2	10	4

15. (a) A company plans to consume 760 pieces of a particular component. Past records indicate that purchasing department had used Rs. 12,000 for placing 15,000 orders. The average inventory was valued at Rs. 45,000 and the total storage cost was Rs. 7,650 which included wages, rent, taxes, insurance, etc., related to store department. The company borrows capital at the rate of 10% a year. If the price of a component is Rs. 12 and the order size is of 10 component, determine; purchase cost, purchase expenses, storage expenses, capital cost and total cost per year.

Or

pages)   **Reg. No. :** ....

Code No. : 20065 E   **Sub. Code : SMMA 53/ AMMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Fifth Semester

Mathematics – Core

STATICS

(For those who joined in July 2017 onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

If the angle between two equal forces P and P is α, then their resultant is ————

a) $2P$

b) $2P\cos\alpha$

c) $2P\cos\frac{\alpha}{2}$

d) 0

2. If the resultant of two forces P and Q is at right angle to P, the angle between the forces is ————

(a) $\cos^{-1}(PQ)$ (b) $\cos^{-1}\left(\frac{P}{Q}\right)$

(c) $\cos^{-1}(-PQ)$ (d) $\cos^{-1}\left(-\frac{P}{Q}\right)$

3. Two paraller forces acting in the same direction are called ———— forces.

(a) Like   (b) Unlike

(c) Direct   (d) Opposite

4. The magnitude of the resultant of two unlike parallel forces is their ————

(a) difference   (b) Sum

(c) multiplication   (d) ratio

5. If three coplanar forces acting on a rigid body keep it in equilibrium, then they must be ————

(a) concurrent   (b) parallel

(c) either (a) or (b)   (d) zero

6. The coefficient of friction $\mu =$

(a) $\frac{F}{R}$ (b) FR

(c) $\tan^{-1}\left(\frac{F}{R}\right)$ (d) $\tan^{-1}(FR)$

The relation between the coefficient of friction and the angle of friction is ————

(a) tan $\mu = \lambda$ (b) tan $\lambda = \mu$

(c) tan $(\lambda\mu) = 1$ (d) $\tan\lambda = \frac{1}{\mu}$

The angle of repose of a rough inclined plane = ————

(a) 0   (b) μ

(c) λ (d) $\tan^{-1}\lambda$

The intrinsic equation of the catenary is ————

(a) $s = c\tan\psi$

(b) $s = \tan\psi$

(c) $s = c\tan\left(\frac{x}{c}\right)$

(d) $s = \tan h\left(\frac{x}{c}\right)$

If the weight per unit length of the chain is constant, then the caternary is called the ———— catenary.

(a) constant   (b) same

(c) common   (d) unique

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the triangle law of forces.

Or

(b) Two forces act on a particle. If the sum and difference of the forces are at right angles to each other, show that the forces are of equal magnitude.

12. (a) Derive the condition of equilibrium of three coplanar paraller forces.

Or

(b) Three like parallel forces, acting at the vertices of a triangle, have magnitudes proportional to the opposite sides. Show that their resultant passes through the incentre of the triangle.

13. (a) State the procedure to be followed in solving any statical problem.

Or

(b) A heavy uniform rod of length 2a rests partly within and partly without a smooth hemispherical bowl of radius r, fixed with its rim horizontal. If α is the inclination of the rod to the horizon, show that $2r\cos 2\alpha = a\cos\alpha$.

[P.T.O.]

14. (a) State the laws of friction.

Or

(b) Write a short note on:

(i) Angle of friction

(ii) Cone of friction

15. (a) Derive the cartesian equation of a catenary.

Or

(b) State and prove any one geometrical property of a common catenary.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Lami's theorem.

Or

(b) If 0 is the circumcentre of the triangle ABC and the forces P,Q,R acting along the lines OA, OB, OC respectively are in equilibrium then prove that

$$P : Q : R = a^2\left(b^2+c^2-a^2\right) : b^2\left(a^2+c^2-b^2\right) : c^2\left(a^2+b^2-c^2\right)$$

17. (a) Force P,Q, R act along the sides BC, AC, BA respectively of an equilateral triangle. If their resultant is a force parallel to BC through the centroid of the triangle, prove that $Q = R = \frac{P}{2}$.

Or

(b) State and prove Varigon's theorem.

18. (a) State and prove two Trigonometical theorems.

Or

(b) A heavy uniform sphere rests touching two smooth inclined planes one of which is inclined at 60° to the horizontal. If the pressure on this plane is one – half of the weight of the sphere, prove that the inclination of the other plane to the horizontal is 30°.

19. (a) A ladder 20 meters long with its centre of gravity 8 meters up from the bottom, weights 60 kg and rests at an angle of θ to the ground against a smooth vertical wall. The coefficient of friction between the ladder and the ground is $\frac{1}{2}$. Find the least value of θ which will enable a weighing 140 kg to reach the top with out the ladder slipping.

Or

(b) A body is at rest on a rough inclined plane and is acted upon by a force parallel to the plane Find the limits between which the force must lie.

20. (a) Find the tension at any point of the catenary.

Or

(b) Explain the parabolic catenary.

———————

(8 pages)

Reg. No. : ....

Code No. : 20066 E **Sub. Code : SMMA 54/ AMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics — Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $F\{f(x)\} = \bar{f}(s)$, then $F\{e^{iax}f(x)\} =$ ________

(a) $\bar{f}(s+a)$ (b) $\bar{f}(x+a)$

(c) $\bar{f}(s-a)$ (d) $\bar{f}(x-a)$

2. $F_S\{f''(x)\} =$ ________

(a) $-sF_s\{f(x)\}+f(0)$

(b) $-s^2F_s\{f(x)\}+sf(0)$

(c) $-sF_c\{f(x)\}+f(0)$

(d) $-s^2F_c\{f(x)\}+sf(0)$

3. If $F\{f(x)\} = \bar{f}(s)$, then $F\{f(ax)\} =$ ________

(a) $\frac{1}{|a|}\bar{f}\left(\frac{s}{a}\right)$ (b) $\bar{f}\left(\frac{s}{a}\right)$

(c) $|a|\bar{f}(sa)$ (d) $\bar{f}(sa)$

4. $\frac{d}{ds}\{F_c(f(x))\} =$ ________

(a) $F_c\{xf(x)\}$ (b) $-F_c\{xf(x)\}$

(c) $F_s\{xf(x)\}$ (d) $-F_s\{xf(x)\}$

5. $F_c\{f(x)\} = \bar{f}_c(n) =$ ________

(a) $\int_0^l f(x)\sin\left(\frac{n\pi.x}{l}\right)dx$

(b) $\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

(c) $\int_0^l f(x)\sin(n\pi x)dx$

(d) $\int_0^l f(x)\cos(n\pi x)dx$

6. $F_c\{f'(x)\} =$ ________

(a) $\frac{-n\pi}{l}\bar{f}_c(n)$

(b) $(-1)^n f(l) - f(0) + \frac{n\pi}{l}\bar{f}_s(n)$

(c) $\frac{n\pi}{l}f_c(n)$

(d) $(-1)^n f(l) + f(0) - \frac{n\pi}{l}\bar{f}_s(n)$

7. $Z(n) =$ ________, where ROC is $|z| > 1$.

(a) $\frac{z}{(z+1)^2}$ (b) $\frac{z(z+1)}{(z-1)^3}$

(c) $\frac{z}{(z-1)^2}$ (d) $\frac{z(z-1)}{(z+1)^2}$

8. $z(a^n) =$ ________, if $|z| > a$.

(a) $\frac{z}{z-a}$ (b) $\frac{z}{z+a}$

(c) $\frac{nz}{z-a}$ (d) $\frac{nz}{z+a}$

9. $z^{-1}\left\{\frac{1}{z+2}\right\} =$ ________

(a) 2^n (b) $(-2)^n$

(c) $(-2)^{n-1}$ (d) 2^{n-1}

10. $z^{-1}\{e^{a/z}\} =$ ________

(a) a^{n-1} (b) a^n

(c) $\frac{a^{n-1}}{(n-1)!}$ (d) $\frac{a_n}{n!}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the Fourier transform of $f(x)$, defined as $f(x) = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin x}{x}dx$.

Or

(b) Find the Fourier transform of $\left\{\frac{\sin ax}{x}\right\}$ and hence prove that $\int_{-\infty}^{\infty} \frac{\sin^2 ax}{x^2} = a\pi$.

[P.T.O.]

12. (a) Find the Fourier cosine transform of e^{-ax} and use it to find the Fourier transform of $e^{-a|x|}\cos bx$.

Or

(b) Find $F_c(e^{-a^2x^2})$.

13. (a) Find the finite Fourier sine and cosine transforms of $\left[1-\frac{x}{\pi}\right]^2$ in $(0, \pi)$.

Or

(b) Find the finite Fourier sine and cosine transforms of e^{ax} in $(0, l)$.

14. (a) Find the z – transform of t^2e^{-t}.

Or

(b) Find the z – transform of $n\cos n\theta$.

15. (a) Find $z^{-1}\left\{\frac{1}{1+4z^{-2}}\right\}$ by the long division method.

Or

(b) Find $z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$, by using Residue theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$, subject to the initial conditions $y(x,0) = f(x)$, $-\infty < x < \infty$. $\frac{\partial y}{\partial t}(x, 0) = g(x)$ and the boundary conditions $y(x, t) \to 0$ as $x \to \pm\infty$.

Or

(b) Solve the equation $(D^2-4D+3)y = \cos 3x$, $x > 0$, given that $y(0) = 0$ and $y'(0) = 0$.

17. (a) Find $f(x)$, if its Fourier sine transform is $\left(\frac{s}{s^2+1}\right)$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$, satisfying the boundary conditions $\frac{\partial u}{\partial x}(0, t) = k$, $t \geq 0$ and $u(x,t) \to 0$ as $x \to 0$ and the initial condition $u(x, 0) = 0$.

18. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, using finite Fourier transforms, given that $u(0,t) = 0$, $u(\pi, t) = 0$, for $t > 0$ and $u(x, 0) = 4\sin^3 x$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, using finite Fourier transform, given that $\frac{\partial u}{\partial x}(0, t) = 0$, $\frac{\partial u}{\partial x}(l, t) = 0$ for $t > 0$ and $u(x,0) = kx$, for $0 < x < l$.

19. (a) Find the z – transform of the following functions

(i) $r^n\cos n\theta$

(ii) $r^n\sin n\theta$

(iii) $\cos n\theta$

(iv) $\sin n\theta$.

Or

(b) Find z – transform of

(i) $f(n) = \frac{1}{n(n-1)}$ and

(ii) $f(n) = \frac{2n+3}{(n+1)(n+2)}$.

20. (a) Find $z^{-1}\left\{\frac{z^2+2z}{z^2+2z+4}\right\}$ by the method of partial fractions.

Or

(b) Solve the simultaneous difference equations. $x_{n+1} = 7x_n + 10y_n$; $y_{n+1} = x_n + 4y_n$, given that $x_0 = 3$ and $y_0 = 2$.

———

(7 pages)

Reg. No. : ....

Code No. : 20067 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The function $f(z) = \bar{z}$ is

(a) differentiable

(b) nowhere differentiable

(c) differentiable only at (0, 0)

(d) none of these

2. If $f(z) = u + iv$ is analytic and $f(z) \neq 0$, then $\nabla^2 amp\ f(z) =$ ————

(a) ∞ (b) 1

(c) $\tan^{-1}\left(\frac{v}{u}\right)$ (d) 0

3. The value of $\int_c \frac{dz}{z-a}$, (c is $|z| = r$) is ————

(a) $2\pi r$ (b) 2π

(c) $2\pi i$ (d) 0

4. The value of $\int_c \frac{dz}{z-3}$, where c is $|z| = 2$

(a) 0 (b) $2\pi i$

(c) $6\pi i$ (d) 1

5. $z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots (-1)^{n-1}\frac{z^n}{n} + \cdots$ represents which of following function?

(a) $\frac{1}{1-z}$ (b) $\log(1+z)$

(c) $\log(1-z)$ (d) $\sin z$

6. ———— is the singular point for the function $ze^{1/z}$

(a) $z = \infty$ (b) $z = 0$

(c) $z = 1$ (d) $z = -\infty$

7. Where we evaluated the $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$, type of integrals?

(a) $|z| > r$ (b) $|z| < r$

(c) $|z| = 1$ (d) $|z| = r$

8. By Jordan's Lemma, the value of $\lim_{r\to\infty} \int_c f(z) e^{iaz} dz =$ ————, where c is the semi-circle.

(a) r (b) $-\infty$

(c) ∞ (d) 0

9. The critical point of the transformation $w = az + b$ is ————

(a) ± 1 (b) a

(c) 0 (d) no critical points

10. Under the transformation $w = iz + 1$, then image of $x > 0$ is ————

(a) $v > 0$ (b) $u > 0$

(c) $-1 < u < 1$ (d) $v < 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Show that $f(z) = \sqrt{r}(\cos\theta/2 + i\sin\theta/2)$, where $r > 0$ and $0 < \theta < 2\pi$ is differentiable.

Or

(b) If $f(z) = u + iv$ is analytic and $f(z) \neq 0$ then prove that $\nabla^2 \log|f(z)| = 0$.

12. (a) Evaluate $\int_c \frac{z+2}{z} dz$, where c is the semi circle $z = 2e^{i\theta}$, where $0 \le \theta \le \pi$.

Or

(b) State and prove Liouville's theorem.

13. (a) Find the Taylor series to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z|<2$.

Or

(b) Use Laurent's series to find the residue of $\frac{d^{2z}}{(z-1)^2}$ at $z=1$.

14. (a) Evaluate $\int_0^{2\pi}\frac{d\theta}{5+4\sin\theta}$.

Or

(b) Evaluate $\int_0^{\infty}\frac{dx}{x^2+1}$.

15. (a) Find the image of the circle $|z-3|=5$ under the transformation $w=\frac{1}{z}$.

Or

(b) Prove that a bilinear transformation $w=\frac{az+b}{cz+d}$, where $ad-bc\neq 0$ maps the real axis into itself iff a,b,c,d are real.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove $C-R$ equations in polar form.

Or

(b) Prove that $u=2x-x^3+3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

17. (a) State and prove Cauchy's theorem.

Or

(b) Evaluate $\int_c \frac{e^z}{(z+2)(z+1)^2}$ where c is $|z|=3$.

18. (a) Expand : $f(z)=\frac{z}{(z-1)(z-2)}$ in a Laurent's series valid for (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$.

Or

(b) Use Cauchy residue theorem, to evaluate $\int_c \frac{3z^2+z-1}{(z^2-1)(z-3)}dz$, around the circle $|z|=2$.

19. (a) Prove that $\int_0^{2\pi}\frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-a^2}}$, $(-1<a<1)$.

Or

(b) Evaluate : $\int_{-\infty}^{\infty}\frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx$, $(a>b>0)$.

20. (a) Find the bilinear transformation which maps $-1, 0, 1$ of the z-plane onto $-1, -i, 1$ of the co-plane.

Or

(b) Prove that a bilinear transformation preserves inverse points.

———————

·.:es)

Reg. No. : ....

·›ae No. : 30344 E **Sub. Code : SMMA 61**

(CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

For those who joined in July 2017 onwards)

Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

f $f(z)=z^2$, then the value of $v(x,y)$

a) x^2-y^2 (b) $2xy$

c) xy (d) x^2+y^2

The complex form of CR equations

a) $f_x=-if_y$ (b) $f_x=if_y$

c) $f_y=-if_x$ (d) $f_x=f_y$

3. If C is the circle with center a and radius r, then the value of $\int_C |z'(t)|\,dt$

(a) $2\pi i$ (b) $-2\pi i$

(c) $2\pi r$ (d) $-2\pi r$

4. If C is the circle $|z-2|=5$, then $\int_C \frac{dz}{z-3}=$ ———

(a) 0 (b) $2\pi i$

(c) $-2\pi i$ (d) πi

5. $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\cdots=?$

(a) $\sin z$ (b) $\cos z$

(c) $\sinh z$ (d) $\cosh z$

6. The poles of $f(z)=\frac{z^2}{(z-2)(z+3)}$

(a) 2, 3 (b) –2, 3

(c) 2, –3 (d) –2, –3

$f(z)=\frac{1}{2z^2+5iz-2}$, then $\operatorname{Res}\{f(z);-i/2\}=?$

) $\frac{1}{3}$ (b) $\frac{1}{3i}$

) $-\frac{1}{3i}$ (d) $-\frac{1}{3}$

he value of $\int_{|z|=2} \tan z\,dz$

) $2\pi i$ (b) $-2\pi i$

) $4\pi i$ (d) $-4\pi i$

he fixed point of the transformation $w=\frac{1}{z-2i}$

) 0 (b) i

) $-i$ (d) $2i$

Which one of the following is not a bilinear ransformation

) $w=z$ (b) $w=\overline{z}$

) $w=1+z$ (d) $w=1-z$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $f(z)=\operatorname{Re} z$ is nowhere differentiable.

Or

(b) If $f(z)$ and $\overline{f(z)}$ are analytic in a region D show that $f(z)$ is constant in that region.

12. (a) Prove that $\int_{-c} f(z)\,dz=-\int_c f(z)\,dz$.

Or

(b) Evaluate $\int_C \frac{z\,dz}{z^2-1}$ where C is the positively oriented circle $|z|=2$.

13. (a) Expand $\cos z$ into a Taylor's series about the point $z=\pi/2$.

Or

(b) Find the residue of $\frac{1}{(z^2+a^2)^2}$ at $z=ai$.

[P.T.O]

14. (a) Evaluate $\int_C \frac{dz}{2z+3}$ where C is $|z| = 2$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

15. (a) Find the bilinear transformation which maps the point $z = -1, 1, \infty$ respectively on $w = -i, -1, i$.

Or

(b) Find the fixed points of the transformation $w = \frac{1+z}{1-z}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive the CR equations in polar co-ordinates.

Or

(b) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

17. (a) State and prove Cauchy's integral formula.

Or

(b) State and prove fundamental theorem of algebra.

18. (a) State and prove Maclaurin's series.

Or

(b) State and prove Cauchy's residue theorem.

19. (a) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1+x^2} dx = \frac{\pi}{2e}$.

20. (a) Find the points where the following mappings are conformal. Also find the critical points if any (i) $w = z^n$ (ii) $w = \frac{1}{2}$.

Or

(b) Prove that a bilinear transformation preserves inverse points.

(6 pages)

Reg. No. : ....

Code No. : 20068 E **Sub. Code : SMMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $\binom{n}{0}$ is ————

(a) 0 (b) n

(c) 1 (d) $n!$

2. Who say "Everything is number"? ————

(a) Pythagoreans (b) Egyptians

(c) Greacean (d) Babylonians

3. gcd (8, 17) is ————

(a) 2 (b) 8

(c) 1 (d) 0

4. If lcm $(a,b)=ab$, then gcd (a,b) is

(a) 0 (b) 1

(c) ab (d) $(ab)^2$

5. The value of $\pi_{4,3}(89)$ is ————

(a) 4 (b) 3

(c) 10 (d) 13

6. Which of the following is twin prime numbers?

(a) 2, 3 (b) 5, 7

(c) 19, 23 (d) 79, 97

7. If $100x \equiv 0(\text{mod}\,3)$, then the value of x is ————

(a) 1 (b) 2

(c) 3 (d) 4

8. Which one of the following is correct? ————

(a) $7 \equiv 0(\text{mod}\,5)$ (b) $7 \equiv 0(\text{mod}\,6)$

(c) $7 \equiv 0(\text{mod}\,7)$ (d) $7 \equiv 0(\text{mod}\,8)$

9. Which one is the smallest pseudoprime to base 3

(a) 91 (b) 217

(c) 341 (d) 561

10. Fermat's theorem says ————

(a) $a^{p-1} \equiv 0(\text{mod}\,p)$

(b) $a^{p-1} \equiv 1(\text{mod}\,p)$

(c) $a^{p-1} \equiv -1(\text{mod}\,p)$

(d) $a^{p} \equiv p-1(\text{mod}\,p)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Archimedean property.

Or

(b) Prove that $\binom{n}{k}+\binom{n}{k-1}=\binom{n+1}{k}$, $1 \le k \le n$.

12. (a) If $a \mid c$ and $b \mid c$ with gcd $(a,b)=1$, then prove that $ab \mid c$.

Or

(b) Prove that any positive integers a and b, gcd (a,b) lcm $(a,b)=ab$.

13. (a) Prove that the number $\sqrt{2}$ is irrational.

Or

(b) Prove that there are an infinite number of primes of the form $4n+3$.

14. (a) Prove that 41 divides $2^{20}-1$.

Or

(b) Solve $9x \equiv 21(\text{mod}\,30)$.

15. (a) State and prove Fermat's theorem.

Or

(b) Using Fermat's method to factorize the number 119143.

[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

using induction.

Or

(b) State and prove binomial theorem.

17. (a) State and prove division algorithm.

Or

(b) Solve the Diophantine equation $24x + 138y = 18$.

18. (a) State and prove fundamental theorem of Arithmetic.

Or

(b) If p_n is the n^{th} prime, then prove that $p_n \leq 2^{2^{n-1}}$.

19. (a) State and prove Chinese remainder theorem.

Or

(b) Explain the Basic properties of congruence.

20. (a) State and prove Wilson's theorem.

Or

(b) Prove that if n is an odd pseudoprime, then $M_n = 2^n - 1$ is a larger one.

(7 pages)

Reg. No. : ....

Code No. : 20069 E **Sub. Code : SMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The maximum degree of a point in a graph with p points is ————

(a) p (b) $p-1$

(c) q (d) p^2

2. If G is a (p, q) graph, then ————

(a) $q \le \binom{p}{2}$ (b) $q = \binom{p}{2}$

(c) $q \ge \binom{p}{2}$ (d) $q = p-1$

3. Which of the following is a graphic sequence?

(a) (1, 1, 1) (b) (2, 2, 1)

(c) (2, 1, 1) (d) (1, 0, 0)

4. The connectivity of the complete graph K_p is ————

(a) p (b) 0

(c) 1 (d) $p-1$

5. Which of the following is an Eulerian graph?

(a) K_6 (b) K_7

(c) $K_{3,3}$ (d) $K_{2,5}$

6. Every Hamiltonian graph is ———— connected.

(a) 2 (b) p

(c) $p-1$ (d) q

7. If G is a maximal planar (p,q) graph, then ————

(a) $q \le 2p-4$ (b) $q \le 3p-6$

(c) $q \ge 3p-6$ (d) $q = 3p-6$

8. $\chi(K_{2,10}) =$

(a) 10 (b) 20

(c) 2 (d) 12

9. If G is a (p,q) graph, then the coefficient of λ^{p-1} in $f(G, \lambda)$ is ————

(a) 0 (b) q

(c) $-q$ (d) p

10. If a complete digraph has n vertices, then it has ———— arcs.

(a) $n(n-1)$ (b) $\frac{n(n-1)}{2}$

(c) $n-1$ (d) $n(n+1)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Prove : $\alpha' + \beta' = p$.

Or

(b) Prove that any self complementary graph has $4n$ or $4n+1$ points.

12. (a) Verify whether the partition (4, 4, 4, 2, 2, 2) is graphical. If it is graphical, draw the corresponding graph.

Or

(b) Prove : A line x of a connected graph G is a bridge if and only if x is not on any cycle of G.

13. (a) If G is a graph with $p \ge 3$ and $\delta \ge \frac{p}{2}$, then show that G is Hamiltonian.

Or

(b) Prove that every tree has a center consisting of either one point or two adjacent points.

14. (a) State prove that Euler's theorem on a connected plane graph.

Or

(b) Show that every uniquely n – colourable graph is $(n-1)$ – connected.

15. (a) Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

Or

(b) Define :

(i) Strongly connected digraph

(ii) Unilaterally connected digraph.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

Or

(b) Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Then prove :

(i) $G_1 \times G_2$ is a (p_1p_2, q_1p_2, q_2p_1) graph

(ii) $G_1[G_2]$ is a $(p_1p_2, p_1q_2, p_2^2q_1)$ graph.

17. (a) State and prove a necessary and sufficient condition for a partition $P = (d_1, d_2, ..., d_p)$ of an even number into parts with $p-1 \geq d_1 \geq d_2 \geq \cdots d_p$ to be graphical.

Or

(b) Show that a graph G with at least two points is bipartite if and only if all its cycles are of even length.

18. (a) Prove that $c(G)$ is well define.

Or

(b) Let G be a (p,q) - graph. Prove that the following are equivalent :

(i) G is a tree

(ii) Every two points of G are joined by a unique path

(iii) G is connected and $p = q + 1$

(iv) G is acyclic and $p = q + 1$.

19. (a) Prove : $\chi'(K_n) = n$, if $n \neq 1$ is odd

$= n - 1$, if n is even.

Or

(b) Show that K_5 and $K_{3,3}$ are non planar graphs.

20. (a) If G is a tree with $n \geq 2$ points, then show that $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.

Or

(b) Prove that the edges of connected graph G can be oriented so that the regulating digraph is strongly connected if and only if every edge of G is contained in at least one cycle.

———————

(6 pages)

Reg. No. : ....

Code No. : 20070 E **Sub. Code : SMMA 64**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

DYNAMICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Time taken by the projectile to reach the greatest height is ————

   (a) $\frac{u\sin\alpha}{g}$ (b) $\frac{u^2\sin\alpha}{g}$

   (c) $\frac{u\sin 2\alpha}{g}$ (d) $\frac{u\sin\alpha}{g^2}$

2. The time of flight of a projectile on an inclined plane is ————

   (a) $\frac{u\sin(\alpha-\beta)}{g}$ (b) $\frac{2u\sin\alpha}{g}$

   (c) $\frac{2u\sin(\alpha-\beta)}{g}$ (d) $\frac{2u\sin(\alpha-\beta)}{g\cos\beta}$

3. The ball is inelastic if

   (a) $v=u$ (b) $v=0$

   (c) $u=\sin\alpha$ (d) $v=1$

4. In Newton's experimental law, the value of e always lies between

   (a) 1 and 2 (b) −1 and 1

   (c) 0 and 1 (d) none of these

5. The equation of simple harmonic motion is

   (a) $\frac{d^2x}{dt^2}=\mu x$ (b) $\frac{d^2x}{dt^2}=-x$

   (c) $\frac{d^2x}{dt^2}=x$ (d) $\frac{d^2x}{dt^2}=-\mu x$

6. The frequency is the reciprocal of ————

   (a) amplitude (b) displacement

   (c) period (d) none

7. The transverse component of velocity is ————

   (a) $\dot{r}$ (b) $r\theta$

   (c) $r\dot{\theta}$ (d) $\dot{r}\dot{\theta}$

8. The polar equation of the equiangular spiral

   (a) $r=e^{\cot\alpha}$ (b) $r=ae^{\cot\alpha}$

   (c) $r=ae^{\theta\cot\alpha}$ (d) $r=\cot\alpha$

9. $(p-r)$ equation of the equiangular spiral is

   (a) $p=\sin\alpha$ (b) $p=r\sin\alpha$

   (c) $p=r$ (d) $p=\cos\alpha$

10. $(p-r)$ equation of the parabola is

    (a) $p=ar$ (b) $p=ar^2$

    (c) $p^2=ar$ (d) $p^2=ar^2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Define a projectile and derive the greatest height attained by it.

    Or

    (b) Derive the range on an inclined plane.

12. (a) Explain the impact of a smooth sphere on a fixed smooth plane.

    Or

    (b) Explain the oblique impact of two smooth spheres.

13. (a) Define simple harmonic motion and derive the equation of motion.

    Or

    (b) Derive the general solution of the simple harmonic motion equation.

14. (a) Explain the equiangular spiral.

    Or

    (b) Derive the radial component of acceleration.

15. (a) Explain the $(p-r)$ equation of the circle.

    Or

    (b) Explain the velocities in a central orbit.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that the path of a projectile is a parabola.

Or

(b) Find the greatest distance of the projectile from the inclined plane and show that is attained in half the total time of flight.

17. (a) Find the loss of kinetic energy due to direct impact of two smooth spheres.

Or

(b) Explain the Newton's experimental law.

18. (a) Find the differential equation of a SHM.

Or

(b) Explain the geometrical representation of simple harmonic motion.

19. (a) Explain the velocity and acceleration in plan coordinates.

Or

(b) Find the differential equation of central orbits.

20. (a) Find the pedal equation of the central orbit.

Or

(b) Find the law of force to an internal point under which a body will describe a circle.

———————

(8 pages)

Reg. No. : ....

Code No. : 20071 E **Sub. Code : SMMA 65**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The positive root of the equation $x^3 - x - 1 = 0$ lies between

   (a) 1 and 2 (b) 0 and 1

   (c) 2 and 3 (d) 3 and 4

2. The order of convergence in Newton - Raphson method is ————

   (a) 3 (b) 2

   (c) 1 (d) 4

3. The value of $\Delta(3^x)$ is ————

   (a) 3^x (b) 3^{x+h}

   (c) $3^x(3^h - 1)$ (d) $3^x - 1$

4. $\Delta(\tan^{-1} x) =$ ————

   (a) $\tan^{-1}\left(\frac{h^2}{1+hx+x^2}\right)$

   (b) $\tan^{-1}\left(\frac{h}{1-hx+x^2}\right)$

   (c) $\tan^{-1}\left(\frac{h}{1+hx-x^2}\right)$

   (d) $\tan^{-1}\left(\frac{h}{1+hx+x^2}\right)$

5. From the following data

x :	5	15	22
y :	7	36	160

   $\Delta\, y_7 =$ ————

   (a) 3.0 (b) 3.1

   (c) 2.9 (d) 2.8

6. When ————, Stirling's formula is used.

   (a) $-\frac{1}{2} < p < \frac{1}{2}$ (b) $p > \frac{1}{2}$

   (c) $p > -\frac{1}{2}$ (d) $-1 < p < 1$

7. The order of the error in Simpson's $\frac{1}{3}$ rule is ————

   (a) h (b) h^2

   (c) h^3 (d) h^4

8. If $f(0) = 1, f(1/3) = 0.75, f(2/3) = 0.6, f(1) = 0.5$, then the value of $\int_0^1 f(x)dx$ using Trapezoidal rule is

   (a) 0.7 (b) 0.6

   (c) 0.8 (d) 1.6

9. The particular integral of $y_{K+2} - 5y_{K-1} + 6y_K = 6^K$ is ————

   (a) 6^{K+1} (b) 6^{K-1}

   (c) $\frac{6^{K-1}}{2}$ (d) $\frac{6^{K+1}}{2}$

10. The order and degree of the equation $y_{x+2} - 3y_{x+1} + 5y_x = x^2$ are ————

    (a) 2, 3 (b) 3, 2

    (c) 3, 3 (d) 2, 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find a real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places by using iteration method.

    Or

    (b) Determine the root of $xe^x - 3 = 0$ correct to three decimal places using the method of false position.

12. (a) Evaluate $\Delta^{10}\left[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)\right]$ if the interval of differencing is 2.

    Or

    (b) Represent the function $x^4 - 12x^3 + 42x^2 - 30x + 9$ and its successive differences in factorial notation where the differencing interval $h = 1$.

[P.T.O.]

13. (a) The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in degree centigrade and p is the percentage of lead in the alloy.

p	40	50	60	70	80	90
t	184	204	226	250	276	304

Using Newton's interpolation formula, find the melting point of the alloy containing 84 percentage of lead.

Or

(b) In the table below, estimate the missing value

x	0	1	2	3	4
y	1	2	4	–	16

Explain why it differs from $2^3 = 8$.

14. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.25$ from the following data.

x	1.00	1.05	1.10	1.15
y	1.00000	1.02470	1.04881	1.07238
x	1.20	1.25	1.30	
y	1.09544	1.11803	1.4017	

Or

(b) Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x\,dx$ by Simpson's $\frac{1}{3}$rd rule.

15. (a) Solve the difference equation $y_{n+1} - 2y_n \cos\alpha + y_{n-1} = 0$.

Or

(b) Eliminate the constants from $y_n = A.2^n + B.3^n$ and derive the corresponding difference equation of the lowest possible order.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Find the inverse of the matrix $\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ by using Gaussian elimination method.

Or

(b) Find the negative root of the equation $x^3 - 2x + 5 = 0$.

17. (a) Prove that

$$y_k = y_0 + k\Delta y_0 + \frac{k(k-1)}{1.2}\Delta^2 y_0 + \cdots + \Delta^k y_0 .$$

Or

(b) Find the second difference of the polynomial $7x^4 + 12x^3 - 6x^2 + 5x - 3$ with interval of differencing $h = 2$.

18. (a) Prove that $y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(P+2)}{3!}\nabla^3 y_n + \cdots + \frac{p(p+1)\cdots(p+n-1)}{n!}\nabla^n y_n$.

Or

(b)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

From this table, find the value of $f(8)$ by using Newton's divided difference formula.

19. (a) Find the Newton's backward difference formula to compute the derivatives.

Or

(b) Using the following data, find $f'(5)$

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

20. (a) Solve the equation

$$y_{n+2} + 2y_{n+1} - 56y_n = 2^n(n^2 - 3).$$

Or

(b) Solve the difference equation $u(x+2) - 4u(x) = 9x^2$.

ges)

Reg. No. : ....

de No. : 30348 E   Sub. Code : SMMA 65

. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017 onwards)

: Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

Order of convergence of Newton's method is

(a) 2   (b) 3

(c) 4   (d) 1

In Gauss – Elimination method the coefficient matrix is converted to a ————

(a) Triangular matrix

(b) Upper triangular matrix

(c) Lower triangular matrix

(d) None of these

3. $\Delta =$ ————

(a) $E-1$ (b) $1-E$

(c) $1+E$ (d) $E^{-1}+1$

4. $\Delta y_0 =$

(a) $y_0 - y_1$ (b) $y_1 - y_0$

(c) $y_2 - y_0$ (d) $y_0 - y_2$

5. In the Gauss forward interpolation formula the value of u is

(a) $\frac{x-x_0}{h!}$ (b) $\frac{x-x_0}{h}$

(c) $\frac{x_0-x}{h}$ (d) $\frac{x+x_0}{h}$

6. The Gauss backward formula involves odd differences ———— the central line.

(a) above   (b) on

(c) below or above   (d) below

The accuracy of the trapezoidal rule can be improved by ————

(a) increasing the number of intervals

(b) increasing the value of h

(c) decreasing the number of intervals

(d) none of these

The error in Simpson's one third rule is of order

(a) h^4 (b) h^5

(c) h^2 (d) linear

The order of $\Delta^2 u_x - 5\Delta u_x - 7u_x = 0$ is ————

(a) 3   (b) 2

(c) 1   (d) 0

The degree of $y_x y_{x+1}^2 - y_{x+2} y_x + 5y_x = x^2 + 7$ is ————

(a) 2   (b) 1

(c) 3   (d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the following system of equations by Gauss Jacobi method.

$8x - 3y + 2z = 20$, $4x + 11y - z = 33$,
$6x + 3y + 12z = 35$.

Or

(b) Find an iterative formula to find $\sqrt{N}$ where N is a +ve.

12. (a) Find the sixth term of the sequence 8, 12, 19, 29, 42,...

Or

(b) Evaluate $\Delta^n (e^{ax} + b)$.

13. (a) Using the following table, apply Gauss's forward formula to get $f(3.75)$.

x :	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	24.145	22.043	20.225	18.644	17.264	16.047

Or

(b) The following table gives some relation between steam pressure and temperature. Find the pressure at temperature 372.1.

T:	361°	367°	378°	387°	399°
P:	154.9	167.9	191.0	212.5	244.2

14. (a) Find $\frac{dy}{dx}$ at the midpoint of

x:	0	300	600	900	1200	1500	1800
y:	135	149	157	183	201	205	193

Or

(b) Evaluate the integral $I = \int_4^{5.2} \log_e x dx$ using Simpson's rule.

15. (a) Form the difference equation given by $y_n (An + B) 3^n$.

Or

(b) Solve $y_{n+1} = \sqrt{y_n}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the approximate root of $x \log_{10} x - 1.2 = 0$ by False position method.

Or

(b) Find the positive root of $f(x) = 2x^3 - 3x - 6 = 0$ by Newton Raphson method correct to 5 decimal places.

17. (a) Prove that

(i) $E\nabla = \Delta = \nabla E$

(ii) $E^{1/2} = \mu + 1/2\delta$

(iii) $\nabla\Delta = \Delta - \nabla = \delta^2$

(iv) $\delta E^{1/2} = \Delta$.

Or

(b) Estimate the production for 1964 and 1966 from the following data.

Year:	1961	1962	1963	1964
Production:	200	220	260	–
Year:	1965	1966	1967	
Production:	350	–	430	

18. (a) From the following table estimate $e^{0.644}$ correct to five decimals using Bessel's formulate. Also find e^x at $x = 0.638$.

x:	0.61	0.62	0.63	0.64
y:	1.840431	1.858928	1.877610	1.896481
x:	0.65	0.66	0.67	
y:	1.915541	1.934792	1.954237	

Or

(b) Use Lagrange's formula to fit a polynomial to the data.

x:	–1	0	2	3
y:	–8	3	1	12

and hence find $y(x = 1)$.

19. (a) Given the following data, find $y'(6)$ and the maximum value of y.

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

Or

(b) By dividing the range into ten equal parts evaluate $\int_0^{\pi} \sin x dx$ by Trapezoidal and Simpson's rule. Verify your answer with integration.

20. (a) Form the Fibanacci difference equation and solve it.

Or

(b) Solve $y_{x+2} - 5y_{x+1} + 6y_x = x^2 + x + 1$.

———————

Pages)

Reg. No. : ....

ode No. : 30357 E Sub. Code : SNMA 4 A/ ANMA 41

(CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics

Non Major Elective — MATHEMATICS FOR COMPETITIVE EXAMINATIONS — II

(For those who joined in July 2017 onwards)

me : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

The money borrowed for a certain period is called ————

(a) principal (b) simple interest

(c) amount (d) interest

2. How much simple interest will Rs. 2000 earn in 18 months at 6% p.a.?

(a) Rs. 120 (b) Rs. 180

(c) Rs. 216 (d) Rs. 240

3. If A's 1 day work = $\frac{1}{n}$, then A can finish the work in ———— days.

(a) 1 (b) 10

(c) n (d) $-n$

4. If A's 1 day work is $\frac{1}{8}$ and B's 1 day work is $\frac{1}{10}$, then $(A+B)$'s 1 day work is ————

(a) 8 (b) 10

(c) $\frac{40}{9}$ (d) $\frac{9}{40}$

5. x Km/hr = ———— m/sec.

(a) $x \times \frac{18}{5}$ (b) $x \times 18$

(c) $x \times \frac{5}{18}$ (d) $x \times 5$

A train is moving with a speed of 180 km/hr. Its speed is

(a) 5 m/sec (b) 30 m/sec

(c) 40 m/sec (d) 50 m/sec

If 15 dolls cost Rs. 35, what do 39 dolls cost?

(a) Rs. 90 (b) Rs. 91

(c) Rs. 89 (d) Rs. 80

The method of finding the 4th proportion when the other three are given is called ———— proportion.

(a) simple (b) compound

(c) direct (d) indirect

A pipe connected with a reservoir, emptying it is known as ————

(a) inlet (b) outlet

(c) full (d) partly full

10. If a pipe can fill a tank in x hours, then part filled in 1 hr = ————

(a) x (b) 0

(c) $\frac{1}{x}$ (d) n

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) At what rate percent per annum will a sum of money double in 8 years?

Or

(b) Find compound interest on Rs. 10000 at 10% p.a. for 2 years and 3 months, compounded annually.

12. (a) A can build a wall in 30 days, while B alone can built it in 40 days. If they build it together and get a payment of Rs. 7000, what is B's share?

Or

(b) A and B together can complete a piece of work in 12 days, B and C can do it in 20 days and C and A can do it in 15 days. A, B and C together can complete it in how many days?

13. (a) Walking at 7/8 of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Or

(b) Tow trains approach each other at 30 km/hr and 27 km/hr from two places 342 km apart. After how many hours will they meet?

14. (a) If 20 men can build a wall 112m long in 6 days, what length of a similar wall can be built by 25 men in 3 days?

Or

(b) 5 men or 9 women can do a piece of work in 19 days. In how many days will 3 men and 6 women do it?

15. (a) Two pipes A and B can fill a tank in 24 hours and 30 hours respectively. If both the pipes are opened simultaneously in the empty tank, how much time will be taken by them to fill it.

Or

(b) A pipe can empty a tank in 40 minutes. A second pipe with diameter twice as much as that of the first is also attached with the tank to empty it. The two together can empty the tank in how much time?

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) What annual instalment will discharge a debt of Rs. 4600 due in 4 years at 10% p.a. simple interest?

Or

(b) The difference between compound interest and simple interest on a certain sum at 8% p.a. for 2 years is Rs. 240. Find the sum.

17. (a) A and B can do a piece of work in 12 days; B and C can do it in 15 days while C and A can do it in 20 days. In how many days will they finish it working together? Also, in how many days can A alone do it?

Or

(b) A can do a piece of work in 10 days while B alone can do it in15 days. They work together for 5 days and the rest of the work is done by C in 2 days and the rest of the work is done by C in 2 days. If they get Rs. 4500 for the whole work, how should they divide the money?

18. (a) A and B are two stations 390 km apart. A train starts form A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

Or

(b) A train traveled distances of 10 km, 20 km and 30 km. At speeds of 50 km/hr, 60 km/hr and 90 km/hr respectively. What is the average speed of the train?

19. (a) If 8 men working 9 hours a day can build a wall 18 m long 2 m broad and 12 m high in 10 days, how many men will be required to build a wall 32 m long, 3 m broad and 9 m high by working 6 hours a day, in 8 days.

Or

(b) 8 women can complete a work in 10 days and 10 children take 16 days to complete the same work. How many days will 10 women and 12 children take to complete the work?

20. (a) A tap can fill the tank in 6 hrs. After half the tank is filled, three more similar taps are opened. What is the total time taken to filled the tank completely?

Or

(b) Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minute more to fill the cistern When the cistern is full, in what time will the leak empty it?

———

(6 pages)

Reg. No. : ....

Code No. : 20077 E **Sub. Code : SSMA 4 A/ ASMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fourth Semester

Mathematics

Skill Based Subject — TRIGONOMETRY, LAPLACE TRANSFORM AND FOURIER SERIES

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $(\cos\theta + i\sin\theta)^n$ is ________

(a) 0 (b) 1

(c) $\cos n\theta + i\sin n\theta$ (d) $\sin n\theta + i\cos n\theta$

2. If $\cos\theta + i\sin\theta = x$, then the value of $x + \frac{1}{x}$ is ________

(a) $2\cos\theta$ (b) $2i\sin\theta$

(c) $2i\cos\theta$ (d) $2\sin\theta$

3. The value of $2\sinh x\cosh x$ is ________

(a) 0 (b) 1

(c) $\cosh 2x$ (d) $\sinh 2x$

4. The value of $\log_e\left(x + \sqrt{x^2+1}\right)$ is ________

(a) $\sinh x$ (b) $\cosh x$

(c) $\sinh^{-1} x$ (d) $\cosh^{-1} x$

5. The value of $L(1)$ is ________

(a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$

(c) $\frac{2}{s^3}$ (d) 0

6. The value of $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ is ________

(a) e^{-at} (b) $e^{-at}t$

(c) $e^{at}t$ (d) e^{at}

7. The value of $L(te^{-at})$ ———

(a) $\frac{1}{s+a}$ (b) $\frac{1}{(s+a)^2}$

(c) $\frac{s}{s+a}$ (d) $\frac{s}{(s+a)^2}$

8. The value of $L^{-1}\left[\frac{2}{(s-a)^3}\right]$ ———

(a) te^{at} (b) t^2e^{at}

(c) te^t (d) t^2e^t

9. The function $\tan x$ is periodic with period ———

(a) 0 (b) 2π

(c) π (d) 3π

10. Which one of the following is an even function?

(a) x (b) x^3

(c) $\sin x$ (d) $e^x + e^{-x}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$.

Or

(b) Expand $\cos^6\theta$ in series of cosines of multiples of θ.

12. (a) Prove that $\sinh^{-1}x = \log_e\left(x + \sqrt{x^2+1}\right)$.

Or

(b) Find $\log(1-i)$.

13. (a) Find $L(\sin^2 2t)$.

Or

(b) Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.

14. (a) Evaluate $\int_0^\infty e^{-2t}\sin 3t\,dt$.

Or

(b) Find $L^{-1}\left[\frac{1+2s}{(s+2)^2(s-1)^2}\right]$.

15. (a) Express $f(x) = x$ as Fourier series in $-\pi < x < \pi$.

Or

(b) Obtain the half range sine series for e^x in $[0, 1]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Prove that $\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$.

Or

(b) Show that

$\sin^3\theta\cos^5\theta =$

$-\frac{1}{2^7(\sin 8\theta + 2\sinh 6\theta 02\sin 4\theta - 6\sin 2\theta)}$

17. (a) If $\cosh u = \sec\theta$, show that $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

Or

(b) Find the general value of $\log_{(-3)}(-2)$.

18. (a) Find (i) $L(\cos at)$ (ii) $L(\sinh at)$.

Or

(b) Prove that $L^{-1}\left[\log\frac{s+1}{s-1}\right] = \frac{2\sinh t}{t}$.

19. (a) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

Or

(b) Solve the equations $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$, $\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$ given $x = 0 = y$ at $t = 0$.

20. (a) Explain the Fourier series for odd and even functions.

Or

(b) Prove that

$|\sin x| = \frac{2}{\pi} - \frac{4}{\pi}\left(\cos\frac{2x}{3} + \frac{\cos 4x}{15} + \cdots\right)$.